ZADACI

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1. Rešiti jednačinu:

$$4^x - 3^{x+\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x+1}$$

2. U pravu kupu poluprečnika osnove $R = 6cm$ upisana je lopta poluprečnika $r = 4cm$. Izračunati visinu H i izvodnicu s te kupe.

3. Naći sva rešenja jednačine:

$$\sin x + \cos x = \frac{1}{\sin x}$$

4. Odrediti jednačinu prave koja odseca na y - osi dva puta veći odsečak nego na x - osi i dodiruje kružnicu.

$$(x-7)^2 + y^2 = 20$$

1. Zadatak

I metoda

$$4^x - s^x \cdot \sqrt{3} = 3^x \cdot \sqrt{3} - 2 \cdot 2^{2x} \Leftrightarrow$$

$$\Leftrightarrow 4^x - \sqrt{3} \cdot 3^x + 2 \cdot \left(2^2\right)^x$$

$$\Leftrightarrow 3 \cdot 4^x - 2 \cdot \sqrt{3} \cdot 3^x = 0 \ / \ : 3 \neq 0 \Leftrightarrow 3 \cdot \left(\frac{4}{3}\right)^x = 2 \cdot \sqrt{3} \ / \ : 3$$

$$\Leftrightarrow \left(\frac{4}{3}\right)^x = \frac{2}{\sqrt{3}} \Leftrightarrow \left(\frac{2}{\sqrt{3}}\right)^{2x} = \frac{2}{\sqrt{3}} \Leftrightarrow 2x = 1 \Leftrightarrow x = \frac{1}{2}$$

1. Zadatak

II metoda

$$3 \cdot 4^x = 2 \cdot \sqrt{3} \cdot 3^x \Leftrightarrow \log\left(3 \cdot 4^x\right) = \log\left(2 \cdot \sqrt{3} \cdot 3^x\right)$$

$$\Leftrightarrow \log 3 + x \cdot \log 4 = \log 2 + \frac{1}{2}\log 3 + x \cdot \log 3$$

$$\Leftrightarrow x \cdot (\log 4 - \log 3) = \log 2 - \frac{1}{2}\log 3 \quad / \cdot 2$$

$$\Leftrightarrow 2x \cdot (\log 4 - \log 3) = 2 \cdot \log 2 - \log 2 - \log 3 \quad / : (\log 4 - \log 3) \neq 0$$

$$\Leftrightarrow 2x = 1 \quad \Leftrightarrow \quad x = \frac{1}{2}$$

2. Zadatak

I Korišćenje sličnosti trouglova

$$\Delta A C_1 \Delta O T C \Rightarrow$$

$$\Rightarrow R : r = H : (S - R) \ \wedge \ R : r = S : (H - r)$$

$$6 : 4 = H : (S - 6) \ \wedge \ 6 : 4 = S : (H - 4)$$

$$6(S - 6) = 4H \ \wedge \ 6(H - 4) = 4S$$

$$6S - 4H = 36 \ \wedge \ 6H - 4S = 24$$

$$3S - 2H = 18 \ \wedge \ -2S + 3H = 12 \Rightarrow$$

$$\Rightarrow H = 14,4CM; \quad S = 15,6CM$$

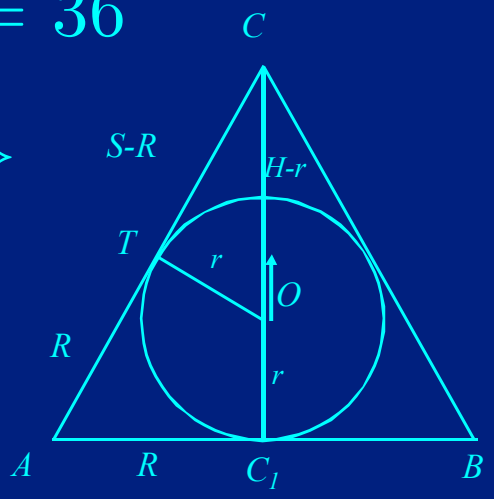
2. Zadatak

I Zadatak se može rešiti i korišćenjem pitagorine teoreme (tehnički ima više računanja)

$$\Delta A C_1 C \Rightarrow S^2 - H^2 = R^2 \Rightarrow S^2 - H^2 = 36$$

$$\Delta O T C \Rightarrow (H - r)^2 - (S - R)^2 = r^2 \Rightarrow$$

$$\Rightarrow (H - 4)^2 = (S - 6)^2 = 16$$



Dobijaju se dve kvadratne jednačine:

$$S^2 - H^2 = 36,\ \ H^2 - S^2 - 8H + 12S = 36 \quad -\text{sabiranjem}$$

$$S^2 - H^2,\ \ -2H + 3S = 18 \quad -\text{rešavanjem se dobija}$$

$$H = \frac{3S - 18}{2},\ \ S^2 - H^2 = 36 \Rightarrow 5S^2 - 6 \cdot S + 18 \cdot 26 =$$

$$S = \frac{3 \cdot 18 \pm 6 \cdot 4}{5} = \frac{78}{5} = 15,6cm \quad H = 14,4$$

3. Zadatak

$$\sin x + \cos x = \frac{1}{\sin x} \Rightarrow \sin x + \cos x - \frac{1}{\sin x} = 0 \Rightarrow$$

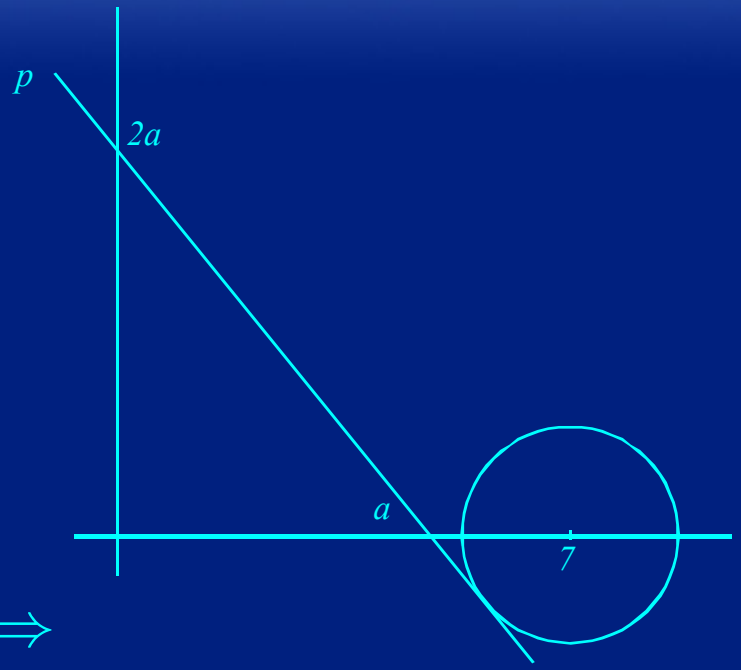
$$\frac{\sin^2 x + \sin x - \cos x - 1}{\sin x} = 0 \Rightarrow$$

$$\frac{\sin^2 x + \sin x \cdot \cos x - \sin^2 x - \cos^2 x}{\sin x} = 0 \Rightarrow$$

$$\frac{\cos x \cdot (\sin x - \cos x)}{\sin x} = 0 \Rightarrow \cos x = 0 \ \vee \ \sin x = \cos x \Rightarrow$$

$$x = (2k+1)\frac{\pi}{2} \ \vee \ x = \frac{\pi}{4} + k\pi, \qquad k = 0, \pm 1, \pm 2, \ldots$$

4. Zadatak



$$(p): \frac{x-a}{0-a} = \frac{y-0}{2a-0}$$

$$(p): y = -2(x-a)$$

$$(x-7)^2 + y^2 = 20$$

$$5x^2 + x(-14-8a) + 4a^2 + 29 = 0 \Rightarrow$$

$$(-14-8a)^2 - 4 \cdot 5\left(4a^2+29\right) = 0 \Rightarrow$$

$$\Rightarrow a^2 - 14a + 24 = 0 \Rightarrow a_1 = 2 \quad a_2 = 12$$

$$(p_1): y = -2(x-2)$$

$$(p_2): y = -2(x-12)$$