Application of Krilov-Bogolubov-Mitropolsky asymptotic methods in hereditary system dynamics

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APPLICATION of KBM ASYMPTOTIC METHODS IN HEREDITARY SYSTEM DYNAMICS

A dynamics of discrete hereditary system is described by an integro-differential form of Lagrange equations :

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} - \sum_{k=1}^{K} b_{kj} \int\limits_0^\infty R_k\left(s\right)\, q_k\left(t-s\right)\, ds \; = \; -\frac{\partial \Pi}{\partial q_j} + Q_j$$

where is a number of hereditary elements in a system, are the relaxation kerns of hereditary element reactions. Let us consider a case where the kerns are presented by fractional-exponential functions (Rhabotnovs functions). Because the hereditary summands enters into the equation (1) with a small parameter the solution of this equation can be found by the asymptotic method of Krilov-Bogolubov-Mitropolsky (KBM methods). For the hereditary mechanical systems with standard rheological bodies the relaxation kerns are presented by the exponential functions. In this case a determination of amplitude-phase functions and entering into resolving equations of KBM asymptotic methods presents no special problems. When the properties of hereditary elements are described by the weakly singular functions the kerns are the weakly singular functions. In this case the finding of abovementioned amplitude-phase functions and carried out by means Euler G-functions. A.V.Rganytsin called attention to the significant difference in results of the averaging of hereditary system parameters over the vibration period at the different stages of motion by virtue of the singularity. To avoid this difference in studies of one-frequency vibration by KBM asymptotic method it is applied original scanning of the averaging limits. For the periodic functions of the variable the segment is used instead of the classical averaging over . In such a manner we can to keep track of the variation of relaxation kern. This scanning averaging allows to replace the value of slowly varied function by its value at the segment and the value of the phase function by a difference function at the averaging segment. The periodic functions and are presented in the complex form. In order to apply the Eulers G-functions these presentations are convenient. Thus the amplitude-phase functions and entering into resolving equations of KBM asymptotic methods have the next form

$$A_1\left(a,\psi,\tau\right) \;=\; -\frac{a\left(\tau\right)}{2\omega\tau}\frac{d\omega}{d\tau}+\frac{a\omega}{2}\,\Im\left\{\int\limits_0^\infty R(s)\,\exp -i\,\omega\,sds\right\}$$

$$B_1\left(a,\psi,\tau\right) \;=\; -\frac{\omega\left(\tau\right)}{2}\,\Re\left\{\int\limits_0^\infty R(s)\,\exp -i\,\omega\,sds\right\}$$

The amplitudes and the phases of the vibrations of hereditary systems are calculated for the some examples. The expressions for the finding of the approximated values of the creep coefficients, the decrements and the frequencies of the hereditary system vibration are presented.

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