INFLUENCE OF LONGITUDINAL INERTIA FORCES ON NONLINEAR FLEXURAL OSCILLATIONS OF BEAMS WITH CONCENTRATED MASSES

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Usually in investigations of nonlinear flexural oscillations of beams there is supposed that the non-linearity is caused by the stretching force appearing due to immovability of the edges of the beam. However there can exist another source of non-linearity – longitudinal displacements and associated with them longitudinal inertia forces. These displacements usually are disregarded since they have the second order of smallness comparing to the transverse ones. But if the beam has concentrated masses (e.g, heavy discs) then it may be anticipated that the longitudinal inertia forces can have an effect upon the nonlinear flexural oscillations.

In this work the influence of the longitudinal inertia on free nonlinear flexural oscillations of beams is studied on the example of a console beam with a disc on the free edge. Equations of motion are derived on the assumptions that the mass of the beam is small comparing to the mass of the disc, and that the beam length remains constant at the flexural oscillations. We take into account possible small difference between the eigenfrequencies in the vertical and horizontal planes (v.p. and h.p.) ω_1 and ω_2, for real (imperfect) beams: $\omega_2 = \omega_1 + \epsilon^2\sigma$ (ϵ is a small parameter, σ is a detuning parameter). Assuming the transverse displacements in the v.p. and h.p. in the form $W(x,\ t) = X_1(x)\theta_1(t), V(x,\ t) = X_2(x)\theta_2(t)$, where $X_1(x)$ and $X_2(x)$ are the first natural modes of bending oscillation (for the beam with the disc) in the v.p. and h.p., respectively, and accounting for the longitudinal displacements of the disc, caused by the bending: $U(t) = -0.5\int_0^l[(\partial W/\partial x)^2 + (\partial V/\partial x)^2]dx$, we reduce the beam to a 2DoF system which dynamics is governed by the following equations with cubic non-linearities:

$$\begin{aligned}\ddot{\theta}_1 + \omega_1^2\theta_1 &= -b\ \theta_1(\dot{\theta}_1^2 + \theta_1\ddot{\theta}_1 + \dot{\theta}_2^2 + \theta_2\ddot{\theta}_2) \\ \ddot{\theta}_2 + (\omega_1^2 + 2\epsilon^2\sigma\omega_1)\theta_2 &= -b\ \theta_2(\dot{\theta}_1^2 + \theta_1\ddot{\theta}_1 + \dot{\theta}_2^2 + \theta_2\ddot{\theta}_2) \qquad (1)\end{aligned}$$

(here b is a generalized parameter depending on the geometry and mass properties of the system). These equations are solved by the multiple scale method [1].

There are obtained equations of amplitude-frequency modulation and their integrals. Stationary (steady-state) and non-stationary motions are studied. It is established that

- due to the longitudinal inertia forces the nonlinear characteristic of the beam becomes soft;
- when the energy of oscillations exceeds a certain threshold value the plane oscillations become unstable;
- simultaneously a spatial ("coupled") oscillation mode appears which is always stable;
- this spatial oscillation mode is an "elliptic" mode [1], [2].

[1]. Nayfeh A. H., Mook D. T. Nonlinear Oscillations. New York, Wiley, 1979, 704 pp.

[2]. Manevich A., Manevich L. The Mechanics of Nonlinear Systems with Internal Resonances. Imperial College Press, 2005, 260 pp.

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