Monograph of Academy of Nonlinear Sciences
Advances in Nonlinear Sciences II - Sciences and Applications, Belgrade
The fractional order hybrid system vibrations, 2008, Vol. 2, pp. 224-324..

THE FRACTIONAL ORDER HYBRID SYSTEM VIBRATIONS

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Summary. *We present some particular solutions of the creeping modes of a system of ordinary fractional order differential equations with analysis of "creeping" modes. By using these particular solutions and modes, a series of engineering dynamical systems are investigated and the same "creeping" modes' different kinds of system dynamics are identified. Main research results presented in this review paper are analytical expressions of modes of three fractional order basic as well as hybrid system vibrations with finite numbers of degrees of freedoms as well as hybrid system containing subsystems with coupled deformable bodies by fractional order distributed standard light elements in the coupling layers. It is shown that two time modes (partial solutions) are pure periodical, and the corresponding number of time modes (particular solutions) are "creeping modes" as results of elastic and /or creeping properties of deformable bodies and also influence of the standard light elements to the periodical mode vibrations with corresponding frequencies.*

Chain dynamics of the homogeneous system – sandwich multi beam, multi plate systems as well as multi pendulum systems are investigated by using mathematical analogy and phenomenological mapping.

Keywords: *Hybrid system, coupled subsystems, coupled dynamics, deformable body, standard light creep element, fractional order derivative, time mode, analytical expression, Laplace transform, "creeping" mode, normal rheolinear modes.*

1 INTRODUCTION

The interest in the study of coupled subsystems, as new qualitative hybrid systems, has grown exponentially over the last few years because of the theoretical challenges involved in the investigation of such systems. A survey as short introduction-review of author's research results in area of dynamics of different kinds of hybrid systems, as well as an analytical approach to the discrete material particle system dynamics containing creep elements described by fractional order derivative, are presented.

Mechanics of hereditary medium (material) is presented in scientific literature by an array of fundamental monographs Rabotnov, Yu.N.[80], Rzhanitsin, A.R. [81], Savin G. N., Ruschisky Yu. Ya [82]. New Analytical mechanics of discrete hereditary systems is presented by O.A.Gorosko and K. (Stevanovic) Hedrih in a monograph [8] and papers [9-12]. Knowledge of Mechanics of hereditary medium as well as Analytical mechanics of discrete hereditary systems is widely used in engineering analyses of strength and deformability of constructions made of new construction materials and presented by different kinds of theoretical models and approaches to solving problems and to obtained analytical expressions of the system dynamics and phenomena dynamics.

These fields of mechanics are being intensively developed and filled up with new research and cited monographs [80-82]. Actuality of that direction of the development of mechanics is conditioned by engineering practice with utilizing new construction materials on synthetic base, the mechanical properties of which often have express creep rheological character.

Nowadays, scale of utilization of these materials can be compared with the scale on which metals are used. New construction materials possess both high strength and different useful physical characteristics: dielectric's properties, radio conductivity, transparentness, high deformability and low (small) weight are what makes them irreplaceable in many cases. Successes of chemistry are enabling production of new synthetic materials with ordered properties.

The university book D.P. Rašković [78] contains the classical theory of longitudinal and transversal oscillations of homogeneous rods and beams, and in [73] we can find mathematical theory of corresponding partial differential equations. R.E.D Bishop's paper [3] contains some results on longitudinal waves in beams.

In two papers[64, 65] by K.S. Hedrih and A. Filipovski, the authors present results of original research on nonlinear and rheolinear oscillations of longitudinal vibrations of an elastic and rheological rod with variable cross section, which has application in engineering systems such as ultrasonic transducers, and ultrasonic concentrator and contain theoretical methods for processing of the vibration state of the ultrasonic concentrator in the form of a rod with variable cross section.

Transversal vibration beam problem is classical, but in current university books on vibrations, we can find only the Euler-Bernoulli's classical partial differential equation for describing transversal beam vibrations. In some references like Ref. [84] we can find a nonlinear partial differential equation for describing transversal vibrations of the beam with nonlinear constitutive stress strain relation of the beam nonlinear ideal elastic material. In the last time period new models of constitutive stress-strain relations of rheological new beam materials [3] can be found in the journal. In the university book [78] of Rašković extended partial differential equations of transversal ideally elastic beam vibration are presented with members by which influences of the inertia rotation of the beam's cross section and shear of the cross section by transversal forces is presented.

Two papers [15,16] by K.S.Hedrih present results on transversal vibrations of prismatic beam of hereditary material. Series of papers [4,5, 6, 15, 18, 19 27 45, 48, 51, 58] by K.S.Hedrih contain new results on transversal vibrations of prismatic beam of the hereditary material or of a fractional derivative order constitutive relation of beam material.

Papers [14, 16, 21, 23, 24, 26, 30, 35, 41, 44, 45, 46, 50, 52, 53, 54-55, 61-64] contain some models of discrete mechanical system dynamics as well as a method of discrete continuum [42] with hereditary light standard element as constraints and with light standard creep element as constraints of the fractional order derivatives in the behaviour of materials [34]. Standard hereditary element is a constraint in systems that are investigated and series of the properties of their dynamics are pointed out and described. Characteristic rheolinear modes with creep properties are expressed by analytical expressions.

Papers [21, 42, 45] contain some models of discrete continuum with hereditary light standard elements as the constraints and with light standard creep element as constraints of the fractional order derivatives in the behaviour of materials. Standard hereditary element is a constraint in the systems which are investigated and described in the monograph [9] as in some of cited papers as used elements.

Integral theory of analytical dynamics of discrete hereditary systems is presented in the monograph [8] and their applications are published in the series of the papers.

In Ref. [7] fractional calculus is mathematically based by corresponding integral and fractional order differential equations.

Refs. [15] and [16] are in relation to transversal vibrations of the beam of the hereditary material and the stochastic stability of the beam dynamic shapes, corresponding to the n-th shape of the beam elastic form.

In paper [1] stochastic stability of viscoelastic systems under bounded noise excitation by Ariaratnam S. T. is investigated. Ref [51] is a contribution on the Ariaratnam idea for investigations of transversal vibration of a parametrically excited beam and influence of rotatory inertia and transverse shear on stochastic stability of deformable forms and processes. Also, Lyapunov exponents are obtained. Asymptotic method of averaging is applied in the previous Reference [51] based on the source monograph by A Mitropolskiy [72-77]. This monograph contains the asymptotic method of averaging generalized for application to nonstationary nonlinear processes.

University books by D.P. Rašković [78, 79] contain the classical theory of strings, beams, plates and shells as well as the longitudinal and transversal oscillation theory of homogeneous rods and beams, as well as partial differential equations of static and dynamic equilibrium of rods with different cross sections, plates and shells.

Transversal vibration plates and shells problem is classical, but in current university books on vibrations, we can find only classical partial differential equations for describing transversal plate vibrations, as in the case of beams and rods. In some monographs we can find a FEM method applied to transversal plate and shell vibrations with nonlinear constitutive stress strain relation of the plates or shells ideal elastic material.

In the last time period new models of constitutive stress-strain relations of the rheological plate new materials can be found in the author's journal papers and monograph papers (see References [20, 12, 23, 27-20, 31, 34, 37, 47, 57, 60, 57, 70]).

A series of previously cited papers by Hedrih contain analytical results of multi plate, multi beam and multi belt system vibrations where plates or beams or belts are coupled by standard light elastic, or hereditary or creep elements distributed between listed deformable bodies.

Some author's initial results of partial fractional order differential equations of creeping and vibrations of plate are presented as a short lecture at ESMC in Thessalonki in 2003, and published in Book of short Abstracts as well as in the sixth pages short journal paper published in Reference [34]. In this Reference [34] a fractional-differential operator with the creep material parameters are introduced. Plate material is creeping and constitutive relation of stress-strain state is expressed through fractional order derivatives. A partial fractional order differential equation of deformed middle surface of the plate has been derived for the case of plate own-free oscillations. For that case, by using a numerical experiment over the solution of the ordinary fractional order differential equation $\ddot{\mathbf{T}}(t)+\left\{\omega_{0mn}^{2}+\omega_{\alpha mn}^{2}\mathfrak{D}_{\mathbf{t}}^{\alpha}\right\}[\mathbf{T}(t)]=0$, time-function surfaces $\mathbf{T}_{mn}(t,\omega_{0mn},\omega_{\alpha mn},\alpha)$ have been constituted as visualizations used for expressing the creeping properties of the plate vibration for some special cases. Generalized results was prepared and submitted for possible publishing. .

In this paper, a complete theoretical approach and series of cases of creep properties of material plate are taken into account and generalized partial fractional order differential equations of transversal vibrations with creeping properties are derived, and analytically solved for series of special cases accompanying by corresponding boundary and initial conditions. These results are new and suitable to be a new additions to classical theory of multi plate system dynamics as well as to be included in the university books for extended and advanced university coerces of dynamics of deformable bodies with different material properties.

2 MODEL OF CREEP RHELOGICAL BODY

By using stress-strain relation from cited References [6, 7, 65, 66], a single-axis stress state of the creep hereditary type material is described by fractional order with respect to time derivative in the form of fractional order differential relation in the form of three parameter model:

$$\sigma(t)=-\left\{E_{0}\varepsilon(t)+E_{\alpha}\mathfrak{D}_{t}^{\alpha}[\varepsilon(t)]\right\} \tag{1}$$

where $\mathfrak{D}_{t}^{\alpha}[\bullet]$ is operator of fractional α^{th} order derivative - the fractional order derivative of strain $\varepsilon(t)$ with respect to time in the following form:

$$\mathfrak{D}_{t}^{\alpha}[\varepsilon(t)]=\frac{d^{\alpha}\varepsilon(t)}{dt^{\alpha}}=\varepsilon^{(\alpha)}(t)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_{0}^{t}\frac{\varepsilon(\tau)}{(t-\tau)^{\alpha}}d\tau \tag{2}$$

where E_0 and E_α are instant and prolonged elasticity modulus, respectively, while α is material relaxation parameter, ratio number from interval $0 < \alpha < 1$, determining fractional order of time derivative, and is Euler gama function. We shall use relation (2) only for $t \geq 0$

2. 1* Longitudinal creep vibrations of a fractional order derivative constitutive relation of the rheological rod with variable cross section

Let us consider a deformable rod of a fractional order derivative constitutive relation of material with variable cross section, whose axis is straight.

Figure 1. shows an element of the rod of variable cross section $A(z)$, where z is axis's length coordinate of the rod. Normal force acting on the cross section at the distance z measured from left side of the rod is:

$$N(z,t) = A(z)\sigma_z(z,t) \tag{3}$$

while it's value in cross section on distance $z + dz$ is:

$$N(z+dz,t) = A(z)\sigma_z(z,t) + \frac{\partial}{\partial z}\left[A(z)\sigma_z(z,t)\right]dz \tag{4}$$

where t is time, and $\sigma_z(z,t)$ is normal stress in the points of cross section that is, according to introduced assumption, invariable on the cross-section. Moreover deplaning of cross section is negligable considering that all points have the same axial displacement determined by coordinate $w(z,t)$, where ρ is rod material's density, and $q(z,t)$ is distributed volume force.

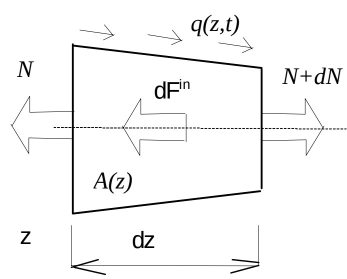


Figure 1. Element of the rod with elementary length dz.

We assume that rod is made of a creep rheological material and therefore the stress-strain-state equation written in the form (1). Taking that strain in axis's:direction of rod is: $\varepsilon_z(z,t) = \frac{\partial w(z,t)}{\partial z}$, previous stress-strain-state relation (1) can be written in following form as:

$$\sigma_z(z,t) = E_0 \frac{\partial w(z,t)}{\partial z} + E_\alpha \mathfrak{D}_t^\alpha\left[\frac{\partial w(z,t)}{\partial z}\right] \tag{6}$$

Introducing previous fractional order derivative stress-strain relation into equilibrium's equation (5), and if we mark $c_0^2 = \frac{E_0}{\rho}$ and $c_\alpha^2 = \frac{E_\alpha}{\rho}$ than previous equation (5) gets the following form:

$$\frac{1}{c_0^2}\frac{\partial^2 w(z,t)}{\partial t^2} - \frac{1}{A(z)}\frac{\partial}{\partial z}\left[A(z)\frac{\partial w(z,t)}{\partial z}\right] = \frac{c_\varepsilon^2}{c_0^2}\frac{1}{A(z)}\frac{\partial}{\partial z}\left[A(z)\mathfrak{D}_t^\alpha\left[\frac{\partial w(z,t)}{\partial z}\right]\right] + \frac{1}{E}q(z,t) \quad (7)$$

2. 2* Free longitudinal creep vibrations of a rod with variable cross section by fractional order derivative in constitutive relation of the rod's material

Solution of the following partial fractional order differential equation:

$$\frac{1}{c_0^2}\frac{\partial^2 w(z,t)}{\partial t^2} - \frac{1}{A(z)}\frac{\partial}{\partial z}\left[A(z)\frac{\partial w(z,t)}{\partial z}\right] = \frac{c_\varepsilon^2}{c_0^2}\frac{1}{A(z)}\frac{\partial}{\partial z}\left[A(z)\mathfrak{D}_t^\alpha\left[\frac{\partial w(z,t)}{\partial z}\right]\right] \quad (8)$$

for free longitudinal creep vibrations of the rod with cross section can be looked for by using Bernoulli's method of particular integrals in the form of multiplication of two functions, from which the first $Z(z)$ depends only on space coordinate z, and the second is time function $T(t)$:

$$w(z,t) = Z(z)T(t) \quad (9)$$

Assumed solution (9) is introduced in previous partial fractional order differential equation (8) and by introducing the constant $\omega_0^2 = k^2 c_0^2$ it is easy to share previous partial fractional order differential equation on following two ordinary differential equations, one of which is fractional order ordinary differential equation:

*first, a second order differential equation on unknown $Z(z)$ eigen function of space coordinate z, with variable coefficients :

$$Z''(z) + \frac{A'(z)}{A(z)}Z'(z) + k^2 Z(z) = 0 \quad (10)$$

and * second, fractional order differential equation on unknown time-function $T(t)$ in the form:

$$\ddot{T}(t) + \omega_\alpha^2 \mathfrak{D}_t^\alpha[T(t)] + \omega_0^2 T(t) = 0 \quad (11)$$

Both differential equations can be solved independently. These are connected only with coupled characteristic constants $\omega_0^2 = k^2 c_0^2$. The first differential equation (10), can be, in some cases, solved for characteristically specified function of variation of cross section of the rod. As it was solved in Refs. [4, 6, 7, 65, 66], for different cases of functions of variation of cross section, in following, we will recall the outcomes from that paper.

3 TRANSVERSAL CREEP VIBRATIONS OF A FRACTIONAL DERIVATIVE ORDER CONSTITUTIVE RELATION HOMOGENEOUS BEAM

For line element of beam creep material, constitutive stress-strain state relation is expressed by fractional order derivative constitutive relation in the form (1), $\sigma_z(z,y,t)$ is normal stress in the point of cross section of the line element, at distance from the left beam end, and at point with distance from neutral axis – bending beam axis, $\varphi(z,t)$ turn angle of beam cross section for pure bending, $\varepsilon_z(z,y,t) = y\dfrac{\partial\varphi(z,t)}{\partial z}$ is dilatation of the line element [69].

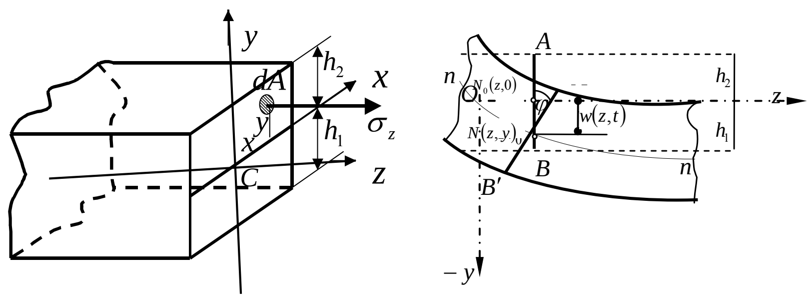


Figure 2. Stress in the beam cross section

The beam transversal displacement is $v(z,t)$, and constitutive relation of the bending couple $\mathfrak{M}_f(z,t)$ is in the following form:

$$\mathfrak{M}_f(z,t) = \left\{ \mathfrak{B}_{0x}\frac{\partial\varphi(z,t)}{\partial z} + \mathfrak{B}_{\alpha x}\mathfrak{D}_t^{\alpha}\left[\frac{\partial\varphi(z,t)}{\partial z}\right]\right\} \tag{12}$$

where $\mathfrak{B}_{0x} = E_0 I_x$ is bending rigidity and $\mathfrak{B}_{\alpha x} = E_\alpha I_x$, fractional bending rigidity of the beam:

Partial-fractional order differential equation of the beam transversal vibrations with respect to the transversal displacement $v(z,t)$ of the beam cross section with distance z from left beam end at the moment t is in the following form:

$$\frac{\partial^2 v(z,t)}{\partial t^2} + \left\{ c_{0x}^2\frac{\partial^4 v(z,t)}{\partial z^4} + c_{\alpha x}^2\mathfrak{D}_t^{\alpha}\left[\frac{\partial^4 v(z,\tau)}{\partial z^4}\right]\right\} -$$

$$- i_x^2\kappa\frac{1}{\mathrm{G}}\left[E_0\frac{\partial^4 v(z,t)}{\partial t^2\partial z^2} + E_\alpha\mathfrak{D}_t^{\alpha}\left[\frac{\partial^4 v(z,t)}{\partial t^2\partial z^2}\right]\right] +$$

$$+i_x^2\kappa\frac{\mathrm{E}_0}{\mathrm{G}}\frac{\partial^4 v(z,t)}{\partial t^4}-i_x^2\frac{\partial^4 v(z,\tau)}{\partial z^2\partial t^2}+\frac{\partial}{\partial z}\left[\mathrm{F}'_N(\Xi,z,t)\frac{\partial v(z,t)}{\partial z}\right]=0 \qquad (13)$$

in which we denoted the following:

$$c_{0x}^2=\frac{\mathfrak{B}_{0x}}{\rho A};c_{\alpha x}^2=\frac{\mathfrak{B}_{\alpha x}}{\rho A};\frac{\mathfrak{B}_{0x}}{\rho A}\frac{\rho\kappa}{G}=\kappa i_x^2\frac{E_0}{G};\frac{\mathfrak{B}_{\alpha x}}{\rho A}\frac{\rho\kappa}{G}=\kappa i_x^2\frac{E_\alpha}{G};$$

$$F_N^{'}(\Xi,z,t)=\frac{1}{\rho A}F_N(\Xi,z,t) \qquad (14)$$

Newly derived partial-fractional order differential equation of the beam transversal vibrations (4) is an extended and generalized equation of transversal vibrations of the beam with members of the creep material properties influence, and the influence of rotation inertia and shear of transversal force. The last member in (13) represents influence of an external force coaxial with beam axis.

From equation (13), we exclude members which contain shear coefficient κ which are in relation to the beam cross section shear under the influence of transversal force, and we suppose that axial forces are equal to zero, and we solve the following equation:

$$\frac{\partial^2 v(z,t)}{\partial t^2}+\left\{c_{0x}^2\frac{\partial^4 v(z,t)}{\partial z^4}+c_{\alpha x}^2\mathfrak{D}_t^\alpha\left[\frac{\partial^4 v(z,\tau)}{\partial z^4}\right]\right\}-i_x^2\frac{\partial^4 v(z,\tau)}{\partial z^2\partial t^2}=0 \quad (13^*)$$

By using Bernoulli's method for solution obtaining, and for solution of partial fractional-differential equation (13*), we can write a product of the two functions depending on separate coordinate z and time t in the following form:

$$v(z,t)=\mathrm{Z}(z)\mathrm{T}(t) \qquad (14)$$

and by introducing the following constants-own beam kinetic parameters ω_{0x}^2 and $\omega_{\alpha x}^2$, which are in the following relations: $\omega_{0x}=k^2c_{0x}=k^2\sqrt{\frac{\mathfrak{B}_{0x}}{\rho\,\mathrm{A}}}$, and $\omega_{\alpha x}=k^2c_{\alpha x}=k^2\sqrt{\frac{\mathfrak{B}_{\alpha x}}{\rho\,\mathrm{A}}}$, with unknown beam transversal vibrations own number k, previous equation (4*) is decomposed to the two equation, one differential with respect to orthogonal normal function $\mathrm{Z}(z)$ of coordinate z, and second time function $\mathrm{T}(t)$ of time t :

$$\mathrm{Z}^{IV}(z)+i_x^2k^4\,\mathrm{Z}''(z)-k^4\,\mathrm{Z}(z)=0 \qquad (15)$$

$$\ddot{\mathrm{T}}(t)+\omega_{\alpha x}^2\mathfrak{D}_t^\alpha[\mathrm{T}(t)]+\omega_{0x}^2\,\mathrm{T}(t)=0$$

3. 1* Transversal creep vibrations of a fractional derivative order constitutive relation of non homogeneous beam

We introduce that material of a one layer beam is a creeping material. Parameters of the beam creep material are: α is proper material constant of the characteristic creep law of material, E_0 and E_α are modulus of elasticity and creeping properties of material.

By using stress-strain relation (1) from cited references, a single-axis stress state of the creep hereditary type material is described by fractional order time derivative differential relation in the form of three parameter model. For line element of beam creep material, constitutive stress-strain state relation is expressed by fractional derivative constitutive relation in the following form:

$$\sigma_z(z,y,t) = y\left\{E_0 \frac{\partial \varphi(z,t)}{\partial z} + E_\alpha \mathfrak{D}_t^\alpha\left[\frac{\partial \varphi(z,t)}{\partial z}\right]\right\} \tag{16}$$

where $\mathfrak{D}_t^\alpha[\bullet]$ is notation of the fractional derivative operator defined by expression (2). $\sigma_z(z,y,t)$ is normal stress in the point of cross section of the line element, at distance z from the left beam end, and at point with distance y from neutral axis – bending beam axis, $\varphi(z,t)$ turn angle of beam cross section for pure bending, $\varepsilon_z(z,y,t) = y\frac{\partial \varphi(z,t)}{\partial z}$ is dilatation of the line element.

In formulation of the problem of stochastic stability of non homogenous creep bars of a fractional derivative order constitutive relation of material is assumed to be continuous function of length coordinate. Let consider the problem on transversal oscillations of two layer straight bar, which is under the action of the length-wise random forces. The excitation processes is a bounded noise excitation.

It is assumed, that layers of the bar were made of creep continuously non homogenous material and the corresponding modulus of elasticity and creep fractional derivative order constitutive relation of the each layer are continuous function of length coordinate and thickness coordinates and changes under the following laws:

$$E_e^{(1)}(z,y) = E_0^{(1)} f_e^{(1)}(z) f_e^{(11)}(y), \quad E_e^{(2)}(z,y) = E_0^{(2)} f_e^{(2)}(z) f_e^{(22)}(y)$$

$$E_\alpha^{(1)}(z,y) = E_{0\alpha}^{(1)} f_\alpha^{(1)}(z) f_\alpha^{(11)}(y), \quad E_\alpha^{(2)}(z,y) = E_{0\alpha}^{(2)} f_\alpha^{(2)}(z) f_\alpha^{(22)}(y)$$

$$0 \le z \le \ell; \quad -h_1 \le y \le h_2 \qquad 0 \le \alpha \le 1 \tag{17}$$

At this case connection between increments of stresses and deformations in each layer represented in a view:

$$\Delta\sigma_z^{(1)} = E_e^{(1)}\Delta\varepsilon_z^{(1)} + E_\alpha^{(1)}\mathfrak{D}_t^\alpha\left[\Delta\varepsilon_z^{(1)}\right] \qquad -h_1 \le y \le 0 \tag{18}$$

$$\Delta\sigma_z^{(2)} = E_e^{(2)}\Delta\varepsilon_z^{(2)} + E_\alpha^{(2)}\mathfrak{D}_t^\alpha\left[\Delta\varepsilon_z^{(2)}\right] \qquad 0 \le y \le h_{2-}$$

Here h_1 and h_2 are thicknesses of the corresponding layers.

Dilatations are:

$$\varepsilon_z = y\frac{\partial\varphi(z,t)}{\partial z} \quad \text{and} \quad \Delta\varepsilon_z = y\frac{\partial\Delta\varphi(z,t)}{\partial z} \tag{19}$$

where $\varphi(z,t)$ is angle of pure bending. normal stress of pure bending is:

$$d\sigma_z^{(1)} = E_0^{(1)} f_e^{(1)}(z) f_e^{(11)}(y)dy\frac{\partial\varphi(z,t)}{\partial z} + E_{0\alpha}^{(1)} f_\alpha^{(1)}(z) f_\alpha^{(11)}(y)\mathfrak{D}_t^\alpha\left[dy\frac{\partial\varphi(z,t)}{\partial z}\right] \tag{20}$$

$$-h_1 \le y \le 0$$

$$d\sigma_z^{(2)} = E_0^{(2)} f_e^{(2)}(z) f_e^{(22)}(y)dy\frac{\partial\varphi(z,t)}{\partial z} + E_{0\alpha}^{(2)} f_\alpha^{(2)}(z) f_\alpha^{(22)}(y)\mathfrak{D}_t^\alpha\left[dy\frac{\partial\varphi(z,t)}{\partial z}\right] \tag{20*}$$

$$0 \le y \le h_{2-}$$

From the dynamic equilibrium conditions we can write:

$$\sum_{i=1}^{N}\vec{F}_i = 0 \quad \text{and} \quad \sum_{i=1}^{N}\vec{M}_0^{\vec{F}_i} = \vec{M}_{fx} = \mathfrak{M}_{fx}^{\vec{F}_i}$$

or

$$\iint_{A''}\sigma_z^{(1)}dxdy + \iint_{A''}\sigma_z^{(2)}dxdy = 0$$

$$\iint_{A''}\sigma_z^{(1)}xdxdy + \iint_{A''}\sigma_z^{(2)}xdxdy \cong 0$$

$$\iint_{A''}\sigma_z^{(1)}ydxdy + \iint_{A''}\sigma_z^{(2)}ydxdy = \mathfrak{M}_{fx} \tag{21}$$

If we introduce following notations:

$$a_\alpha^{(1)(n)} = \int_{-h_1}^{0} f_\alpha^{(11)}(y)y^n dy,\ a_\alpha^{(2)(n)} = \int_{0}^{h_2} f_\alpha^{(22)}(y)y^n dy,\ n = 0,1,2 \tag{22}$$

previous equilibrium conditions we can write in the following relations: and expression:

$$a_e^{(1)(1)} E_0^{(1)} f_e^{(1)}(z) - a_e^{(2)(1)} E_0^{(2)} f_e^{(2)}(z) = 0 \Rightarrow f_e^{(2)}(z) = f_e^{(1)}(z) \frac{E_0^{(1)}}{E_0^{(2)}} \frac{a_e^{(1)(1)}}{a_e^{(2)(1)}} \quad (23)$$

$$a_\alpha^{(1)(1)} E_{0\alpha}^{(1)} f_\alpha^{(1)}(z) - a_\alpha^{(2)(1)} E_{0\alpha}^{(2)} f_\alpha^{(2)}(z) = 0 \Rightarrow f_\alpha^{(2)}(z) = f_\alpha^{(1)}(z) \frac{E_{0\alpha}^{(1)}}{E_{0\alpha}^{(2)}} \frac{a_\alpha^{(1)(1)}}{a_\alpha^{(2)(1)}} \quad (24)$$

and following expression for bending moment:

$$\begin{aligned} \mathfrak{M}_{fx}(z,t) &= b \frac{\partial \varphi(z,t)}{\partial z} \left\{ E_0^{(1)} a_e^{(1)(2)} f_e^{(1)}(z) + E_0^{(2)} a_e^{(2)(2)} f_e^{(2)}(z) \right\} + \\ &+ b \mathfrak{D}_t^\alpha \left[\frac{\partial \varphi(z,t)}{\partial z} \right] \left\{ E_{0\alpha}^{(1)} a_\alpha^{(1)(2)} f_\alpha^{(1)}(z) + E_{0\alpha}^{(2)} a_\alpha^{(2)(2)} f_\alpha^{(2)}(z) \right\} \end{aligned} \quad (25)$$

or with respect to the previous relations (22), (23) and (24) we can write in the following form:

$$\begin{aligned} \mathfrak{M}_{fx}(z,t) &= E_0^{(1)} b \frac{\partial \varphi(z,t)}{\partial z} f_e^{(1)}(z) \left\{ a_e^{(1)(2)} + a_e^{(2)(2)} \frac{a_e^{(1)(1)}}{a_e^{(2)(1)}} \right\} + \\ &+ E_{0\alpha}^{(1)} b f_\alpha^{(1)}(z) \mathfrak{D}_t^\alpha \left[\frac{\partial \varphi(z,t)}{\partial z} \right] \left\{ a_\alpha^{(1)(2)} + a_\alpha^{(2)(2)} \frac{a_\alpha^{(1)(1)}}{a_\alpha^{(2)(1)}} \right\} \end{aligned} \quad (26)$$

We take into account the rotatory inertia of cross section and we can write the following equations of bar dynamics:

$$d\,\mathrm{J}_x \frac{\partial^2 \varphi(z,t)}{\partial t^2} = -d\mathfrak{M}_f(z,t) + \mathrm{F}_T(z,t) dz + \mathrm{F}_N(\Xi, z, t) dv(z,t) \quad (27)$$

$$dm \frac{\partial^2 v(z,t)}{\partial t^2} = d\,\mathrm{F}_T(z,t) \quad (28)$$

If we introduce:

$$dm = (\rho_1 A_1 + \rho_2 A_2) dz \quad \text{and} \quad d\mathbf{J}_x = \left[\rho_1 \mathbf{I}_\mathbf{x}^{(1)} + \rho_2 \mathbf{I}_\mathbf{x}^{(2)} \right] dz \quad (29)$$

we can write:

$$\begin{aligned} \left[\rho_1 \mathbf{I}_x^{(1)} + \rho_2 \mathbf{I}_x^{(2)} \right] \frac{\partial^2 \varphi(z,t)}{\partial t^2} &= E_0^{(1)} b \left[a_e^{(1)(2)} + a_e^{(2)(2)} \frac{a_e^{(1)(1)}}{a_e^{(2)(1)}} \right] \frac{\partial}{\partial z} \left[\frac{\partial \varphi(z,t)}{\partial z} f_e^{(1)}(z) \right] + \\ &+ E_{0\alpha}^{(1)} b \left[a_\alpha^{(1)(2)} + a_\alpha^{(2)(2)} \frac{a_\alpha^{(1)(1)}}{a_\alpha^{(2)(1)}} \right] \frac{\partial}{\partial z} \left\{ f_\alpha^{(1)}(z) \mathfrak{D}_t^\alpha \left[\frac{\partial \varphi(z,t)}{\partial z} \right] \right\} + \mathbf{F}_T + F_N \frac{\partial v(z,t)}{\partial z} \end{aligned} \quad (30)$$

$$`\left(\rho_1 A_1+\rho_2 A_2\right)\frac{\partial^2 v(z,t)}{\partial t^2}=\frac{\partial F_T(z,t)}{\partial z} \qquad (31)$$

After applying derivative with respect to time we obtain the following partial-fractional differential equation:

$$\frac{\partial^2 v(z,t)}{\partial t^2}+\frac{E_0^{(1)}b\left[a_e^{(1)(2)}+a_e^{(2)(2)}\frac{a_e^{(1)(1)}}{a_e^{(2)(1)}}\right]}{\left(\rho_1 A_1+\rho_2 A_2\right)}\frac{\partial^2}{\partial z^2}\left[\frac{\partial^2 v(z,t)}{\partial z^2}f_e^{(1)}(z)\right]+\frac{1}{\left(\rho_1 A_1+\rho_2 A_2\right)}\frac{\partial}{\partial z}\left[F_N\frac{\partial v(z,t)}{\partial z}\right]+ \qquad (32)$$
$$+\frac{E_{0\alpha}^{(1)}b\left[a_\alpha^{(1)(2)}+a_\alpha^{(2)(2)}\frac{a_\alpha^{(1)(1)}}{a_\alpha^{(2)(1)}}\right]}{\left(\rho_1 A_1+\rho_2 A_2\right)}\frac{\partial^2}{\partial z^2}\left\{f_\alpha^{(1)}(z)\mathfrak{D}_t^\alpha\left[\frac{\partial^2 v(z,t)}{\partial z^2}\right]\right\}+\frac{\left[\rho_1\mathbf{I}_x^{(1)}+\rho_2\mathbf{I}_x^{(2)}\right]}{\left(\rho_1 A_1+\rho_2 A_2\right)}\frac{\partial^4 v(z,t)}{\partial t^2\partial z^2}=0$$

By introducing following notations:

$$\tilde{c}_{0x}^2=\frac{\tilde{E}_0^{(1)}}{\rho}\tilde{i}_{xe}^2=\frac{E_0^{(1)}b\left[a_e^{(1)(2)}+a_e^{(2)(2)}\frac{a_e^{(1)(1)}}{a_e^{(2)(1)}}\right]}{\left(\rho_1 A_1+\rho_2 A_2\right)},\quad \tilde{i}_x^2=\frac{\left[\rho_1\mathbf{I}_x^{(1)}+\rho_2\mathbf{I}_x^{(2)}\right]}{\left(\rho_1 A_1+\rho_2 A_2\right)},$$
$$\tilde{c}_{0x\alpha}^2=\frac{\tilde{E}_{0\alpha}^{(1)}}{\rho}\tilde{i}_{x\alpha}^2=\frac{E_{0\alpha}^{(1)}b\left[a_\alpha^{(1)(2)}+a_\alpha^{(2)(2)}\frac{a_\alpha^{(1)(1)}}{a_\alpha^{(2)(1)}}\right]}{\left(\rho_1 A_1+\rho_2 A_2\right)},\quad \hat{i}_x^2=\frac{\left[\rho_1\mathbf{I}_x^{(1)}+\rho_2\mathbf{I}_x^{(2)}\right]}{A\rho},$$
$$\tilde{i}_{xe}^2=\frac{b\left[a_e^{(1)(2)}+a_e^{(2)(2)}\frac{a_e^{(1)(1)}}{a_e^{(2)(1)}}\right]}{A},\quad \tilde{i}_{x\alpha}^2=\frac{b\left[a_\alpha^{(1)(2)}+a_\alpha^{(2)(2)}\frac{a_\alpha^{(1)(1)}}{a_\alpha^{(2)(1)}}\right]}{A}. \qquad (33)$$

we obtain the following partial-fractional differential equation of transversal vibrations of creep of two layer straight bar, which is under the action of the length-wise random forces:

$$\frac{\partial^2 v(z,t)}{\partial t^2}+\tilde{c}_{0x}^2\frac{\partial^2}{\partial z^2}\left[\frac{\partial^2 v(z,t)}{\partial z^2}f_e^{(1)}(z)\right]+\frac{1}{\left(\rho_1 A_1+\rho_2 A_2\right)}\frac{\partial}{\partial z}\left[F_N\frac{\partial v(z,t)}{\partial z}\right]+ \qquad (34)$$
$$+\tilde{c}_{0x\alpha}^2\frac{\partial^2}{\partial z^2}\left\{f_\alpha^{(1)}(z)\mathfrak{D}_t^\alpha\left[\frac{\partial^2 v(z,t)}{\partial z^2}\right]\right\}-\tilde{i}_x^2\frac{\partial^4 v(z,t)}{\partial t^2\partial z^2}=0$$

We study a special case: From equation (34), we exclude members which contain axial forces and we solve the following equation:

$$\frac{\partial^2 v(z,t)}{\partial t^2}+\tilde{c}_{0x}^2\frac{\partial^2}{\partial z^2}\left[\frac{\partial^2 v(z,t)}{\partial z^2}f(z)\right]+\tilde{c}_{0x\alpha}^2\frac{\partial^2}{\partial z^2}\left\{f(z)\mathfrak{D}_t^\alpha\left[\frac{\partial^2 v(z,t)}{\partial z^2}\right]\right\}-\tilde{i}_x^2\frac{\partial^4 v(z,t)}{\partial t^2\partial z^2}=0 \qquad (35)$$

where

$$f_e^{(1)}(z)=f_\alpha^{(1)}(z)=f(z) \qquad (36)$$

By using Bernoulli's method for solution obtaining, and for solution of a partial fractional-differential equation (35), we can write a product of the two functions depending on separate coordinate z and time t in the following form:

$$v(z,t)=Z(z)T(t) \tag{37}$$

By introducing this solution into equation (37) we obtain two equations:

$$\frac{d^2}{dz^2}\left[Z''(z)f(z)\right]+\tilde{i}_x^{\,2}k^4Z''(z)-k^4Z(z)=0$$

$$\ddot{\mathrm{T}}(t)+\tilde{\omega}_{\alpha x}^2\mathfrak{D}_t^\alpha\left[\mathrm{T}(\mathrm{t})\right]+\tilde{\omega}_{0x}^2\,\mathrm{T}(t)=0 \tag{38}$$

where

$$\tilde{\omega}_{0x}^2=k^4\tilde{c}_{0x}^2=k^4\frac{E_0^{(1)}b\left[a_e^{(1)(2)}+a_e^{(2)(2)}\dfrac{a_e^{(1)(1)}}{a_e^{(2)(1)}}\right]}{\left(\rho_1A_1+\rho_2A_2\right)}$$

$$\tilde{\omega}_{\alpha x}^2=k^4\tilde{c}_{0x\alpha}^2=k^4\frac{E_{0\alpha}^{(1)}b\left[a_\alpha^{(1)(2)}+a_\alpha^{(2)(2)}\dfrac{a_\alpha^{(1)(1)}}{a_\alpha^{(2)(1)}}\right]}{\left(\rho_1A_1+\rho_2A_2\right)} \tag{39}$$

$$\tilde{i}_x^{\,2}=\frac{\left[\rho_1\mathbf{I}_x^{(1)}+\rho_2\mathbf{I}_x^{(2)}\right]}{\left(\rho_1A_1+\rho_2A_2\right)}$$

3. 2* The time function solution of a fractional order differential equations

The second, fractional-differential equation from all three considered cases is mathematically same fractional-differential equation with unknown time-function $T(t)$ and we can rewrite it in the following form:

$$\ddot{T}(t)+\omega_\alpha^2T^{(\alpha)}(t)+\omega_0^2T(t)=0 \tag{44}$$

This fractional-differential equation (40) on unknown time-function $T(t)$, can be solved applying Laplace transforms (see Ref. [8], [9], [6], [18] and [15]). Upon that fact Laplace transform of solution is in the form:

$$\mathfrak{T}(p)=\mathfrak{L}[T(t)]=\frac{pT(0)+\dot{T}(0)}{p^2+\omega_0^2\left[1+\dfrac{\omega_\alpha^2}{\omega_0^2}\mathbf{R}(p)\right]} \tag{41}$$

where $\mathfrak{L}\left[\mathfrak{D}_t^\alpha[T(t)]\right]=\mathbf{R}(p)\mathfrak{L}[\mathrm{T}(\mathrm{t})]$ is Laplace transform of a fractional derivative $\dfrac{d^\alpha T(t)}{dt^\alpha}$ for $0\le\alpha\le1$. For creep rheological material those Laplace transforms the form:

$$\mathfrak{L}\left[\mathfrak{D}_t^\alpha[T(t)]\right]=\mathbf{R}(p)\mathfrak{L}[T(t)]-\frac{d^{\alpha-1}}{dt^{\alpha-1}}T(0)=p^\alpha\mathfrak{L}[T(t)]-\frac{d^{\alpha-1}}{dt^{\alpha-1}}T(0) \tag{42}$$

where the initial value are:

$$\left.\frac{d^{\alpha-1}T(t)}{dt^{\alpha-1}}\right|_{t=0}=0 \tag{42*}$$

so, in that case Laplace transform of time-function is given by following expression:

$$\mathfrak{L}\{T(t)\}=\frac{pT_0+\dot{T}_0}{\left[p^2+\omega_\alpha^2 p^\alpha+\omega_0^2\right]} \tag{43}$$

For boundary cases, when material parameters α take following values: $\alpha=0$ i $\alpha=1$ we have two special simple cases, whose corresponding fractional-differential equations and solutions are known. In these cases fractional-differential equations are:

$$1^*\quad \ddot{T}(t)+\tilde{\omega}_0^2 T^{(0)}(t)+\omega_0^2 T(t)=0 \quad \text{for} \quad \alpha=0 \tag{44}$$

where $T^{(0)}(t)=T(t)$, and

$$2^*\quad \ddot{T}(t)+\omega_1^2 T^{(1)}(t)+\omega_0^2 T(t)=0 \quad \text{for} \quad \alpha=1 \tag{45}$$

where $T^{(1)}(t)=\dot{T}(t)$.

The solutions to equations (44) and (45) are:

$$1^*\quad T(t)=T_0\cos t\sqrt{\omega_0^2+\tilde{\omega}_0^2}+\frac{\dot{T}_0}{\sqrt{\omega_0^2+\tilde{\omega}_0^2}}\sin t\sqrt{\omega_0^2+\tilde{\omega}_0^2} \tag{46}$$

For $\alpha=0$;

$$2^*\text{ a.}\quad T(t)=e^{-\frac{\omega_1^2}{2}t}\left\{T_0\cos t\sqrt{\omega_0^2-\frac{\omega_1^4}{4}}+\frac{\dot{T}_0}{\sqrt{\omega_0^2-\frac{\omega_1^4}{4}}}\sin t\sqrt{\omega_0^2-\frac{\omega_1^4}{4}}\right\} \tag{47}$$

for $\alpha=1$ and for $\omega_0>\frac{1}{2}\omega_1^2$. (for soft creep) or for strong creep:

$$2^*\text{ b.}\quad T(t)=e^{-\frac{\omega_1^2}{2}t}\left\{T_0 Ch\, t\sqrt{\frac{\omega_1^4}{4}-\omega_0^2}+\frac{\dot{T}_0}{\sqrt{\frac{\omega_1^4}{4}-\omega_0^2}}Sh\, t\sqrt{\frac{\omega_1^4}{4}-\omega_0^2}\right\} \tag{48}$$

for $\alpha=1$ and for $\omega_0<\frac{1}{2}\omega_1^2$.

For kritical case:

$$2^*\text{ c.}\quad T(t)=e^{-\frac{\omega_1^2}{2}t}\left\{T_0+\frac{2\dot{T}_0}{\omega_1^2}t\right\}\text{ za } \alpha=1 \quad \text{and za} \quad \omega_0=\frac{1}{2}\omega_1^2. \tag{49}$$

Fractional-differential equation (40) for the general case, when α is real number from interval $0<\alpha<1$ can be solved by using Laplace's transformation. By using that is:

$$\mathfrak{L}\left\{\frac{d^{\alpha}T(t)}{dt^{\alpha}}\right\}=p^{\alpha}\mathfrak{L}\{T(t)\}-\left.\frac{d^{\alpha-1}T(t)}{dt^{\alpha-1}}\right|_{t=0}=p^{\alpha}\mathfrak{L}\{T(t)\} \tag{50}$$

and by introducing initial conditions of fractional derivatives in the form (42*), and after taking Laplace's transform of the equation (40) we obtain the Laplace transform of solution in the form (41).

By analyzing previous Laplace transform (50) of solution we can conclude that we can consider two cases.

For the case when $\omega_0^2 \neq 0$, the Laplace transform solution can be developed into series by following way:

$$\mathfrak{L}\{T(t)\}=\frac{pT_0+\dot{T}_0}{p^2\left[1+\frac{\omega_\alpha^2}{p^2}\left(p^{\alpha}+\frac{\omega_0^2}{\omega_\alpha^2}\right)\right]}=\left(T_0+\frac{\dot{T}_0}{p}\right)\frac{1}{p}\frac{1}{1+\frac{\omega_\alpha^2}{p^2}\left(p^{\alpha}+\frac{\omega_0^2}{\omega_\alpha^2}\right)} \tag{51}$$

$$\mathfrak{L}\{T(t)\}=\left(T_0+\frac{\dot{T}_0}{p}\right)\frac{1}{p}\sum_{k=0}^{\infty}\frac{(-1)^k\omega_\alpha^{2k}}{p^{2k}}\left(p^{\alpha}+\frac{\omega_0^2}{\omega_\alpha^2}\right)^k \tag{52}$$

$$\mathfrak{L}\{T(t)\}=\left(T_0+\frac{\dot{T}_0}{p}\right)\frac{1}{p}\sum_{k=0}^{\infty}\frac{(-1)^k\omega_\alpha^{2k}}{p^{2k}}\sum_{j=0}^{k}\binom{k}{j}\frac{p^{\alpha j}\omega_\alpha^{2(j-k)}}{\omega_o^{2j}} \tag{53}$$

In writing (53) it is assumed that expansion leads to convergent series [9]. The inverse Laplace transform of previous Laplace transform of solution (53) in term-by-term steps is based on known theorem, and yield the following solution of differential equation (40) of time function in the following form of time series:

$$\begin{aligned}T(t)=\mathfrak{L}^{-1}\{T(t)\}=T_0\sum_{k=0}^{\infty}(-1)^k\omega_\alpha^{2k}t^{2k}\sum_{j=0}^{k}\binom{k}{j}\frac{\omega_\alpha^{2j}t^{-\alpha j}}{\omega_o^{2j}\Gamma(2k+1-\alpha j)}+\\+\dot{T}_0\sum_{k=0}^{\infty}(-1)^k\omega_\alpha^{2k}t^{2k+1}\sum_{j=0}^{k}\binom{k}{j}\frac{\omega_\alpha^{-2j}t^{-\alpha j}}{\omega_o^{2j}\Gamma(2k+2-\alpha j)}\end{aligned} \tag{54}$$

or

$$T(t)=\mathfrak{L}^{-1}\{T(t)\}=\sum_{k=0}^{\infty}(-1)^k\omega_\alpha^{2k}t^{2k}\sum_{j=0}^{k}\binom{k}{j}\frac{\omega_\alpha^{2j}t^{-\alpha j}}{\omega_o^{2j}}\left[\frac{T_0}{\Gamma(2k+1-\alpha j)}+\frac{\dot{T}_0 t}{\Gamma(2k+2-\alpha j)}\right] \tag{55}$$

Two special cases of the solution for $\omega_0^2=0$ are:

$$T(t)=T_0\cos\tilde{\omega}_o t+\frac{\dot{T}_0}{\tilde{\omega}_0}\sin\tilde{\omega}_o t \quad \text{for } \alpha=0 \text{ and } \omega_0^2=0. \tag{56}$$

$$T(t) = T_0 + \frac{\dot{T}_0}{\omega_1^2}\left(1 - e^{-\omega_1^2 t}\right) \qquad \text{for } \alpha = 1 \quad \text{and} \quad \omega_0^2 = 0 \tag{57}$$

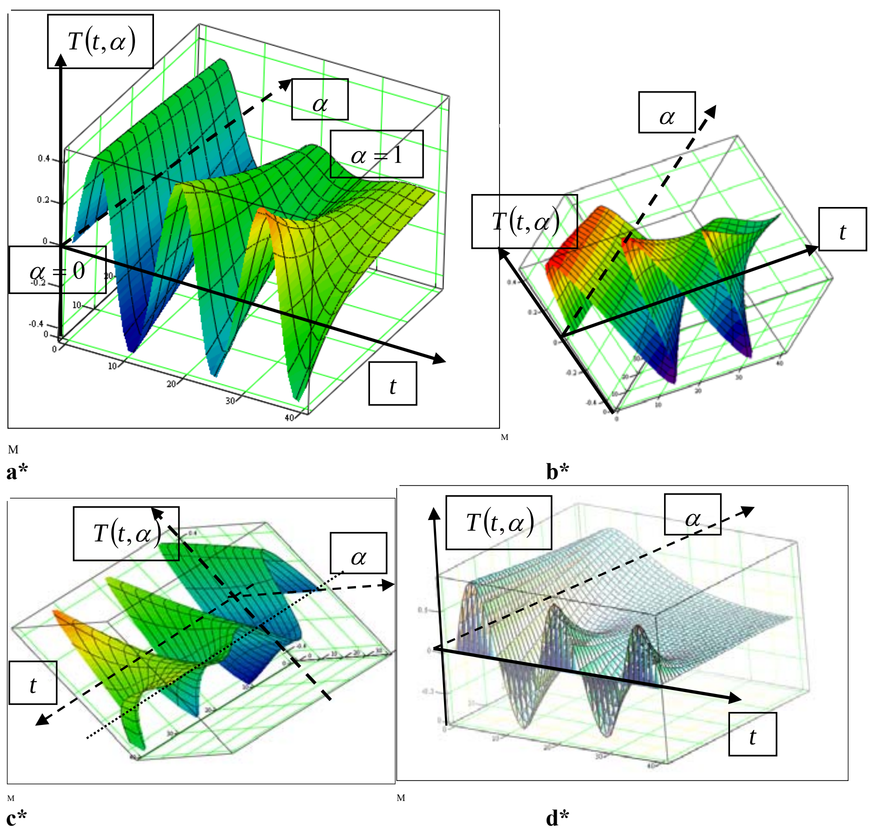


Figure 3. Numerical simulations and graphical presentation of the results. Time functions $T(t,\alpha)$ surface for the different beam transversal vibrations kinetic and creep material parameters:

a* $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right) = 1$; **b*** $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right) = \frac{1}{4}$; **c*** $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right) = \frac{1}{3}$; **d*** $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right) = 3$.

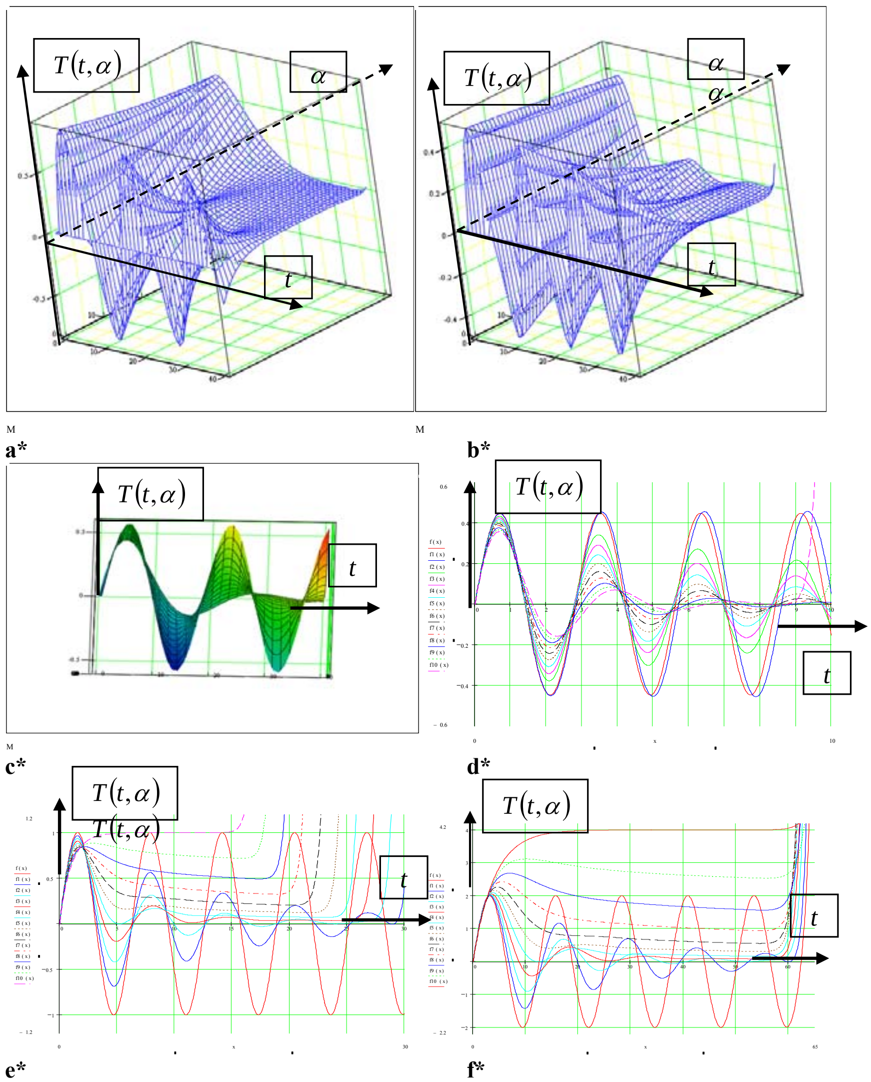


Figure 4. Numerical simulations and graphical presentation of the results. Time functions $T(t,\alpha)$ surface and curves families for the different beam transversal vibrations kinetic and discrete values of the creep material parameters $0 \le \alpha \le 1$:
a* and **c*** $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right) = 1$; b* and **d*** $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right) = \frac{1}{4}$; **e*** $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right) = \frac{1}{3}$; **f*** $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right) = 3$.

In Figure 3. numerical simulations and graphical presentation of the solution of the fractional-differential equation of the system (27*) are presented. Time functions $T(t,\alpha)$ surfaces for the different beam transversal vibrations kinetic and creep material parameters in the space $(T(t,\alpha),t,\alpha)$ for interval $0 \le \alpha \le 1$ are visible:

in **a*** for $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right)=1$; in **b*** for $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right)=\frac{1}{4}$; in **c*** for $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right)=\frac{1}{3}$; in **d*** for $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right)=3$.

In Figure 4. time functions $T(t,\alpha)$ surfaces and curves families for the different beam transversal vibrations kinetic and discrete values of the creep material parameters $0 \le \alpha \le 1$ are presented. In Figures **a*** and **c*** for $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right)=1$; in Figures b* and **d*** for $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right)=\frac{1}{4}$; in Figure **e*** for $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right)=\frac{1}{3}$; and in Figure **f*** for $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right)=3$.

4 PARTIAL FRACTIONAL ORDER DIFFERENCIAL EQUATIONS OF CREEPING AND VIBRATIONS OF PLATE

4. 1* Basic suppositions (presumptions) of the cinematic deformation of a platE.

Let's introduce (suppose) that plate is thin and that there is no depalanation of cross sections in conditions of creep material. Also, we suppose that cross sections are always orthogonal with respect to the middle plane of the plate. If a thin plate is creep bent with small deflection, i.e., when the deflection of the middle surface is small compared to the thickness h, the following assumption can be made:

1* The normal to middle surface before creep bending are deformed into normals of the middle surface after bending.

2* The stress σ_z is small compared with the other stress components and may be neglected in the stress strain relations. 3* The middle surface remains unstrained after bending.

On the basis of the previous, we suppose that displacements $u(x,y,z,t)$ and $v(x,y,z,t)$ of the point $N(x,y,z)$ in the direction of the coordinate axes x and y are possible to express in the function of its distance z from plate middle surface and its transversal displacement $w(x,y,t)$ in direction of the axis z, and also same displacement of the corresponding point $N_0(x,y,0)$ in the plate middle surface. By using method from Ref. [78,79] (see D. Rašković), we can write the expression for displacement of the plate point $N(x,y,z)$ in the following form:

$$u = -ztg\alpha \approx -z\alpha = -z\frac{\partial w}{\partial x}$$
$$v = -ztg\beta \approx -z\beta = -z\frac{\partial w}{\partial y} \qquad (58)$$

Curvatures of the plate are:

$$\mathbf{K}_x = \frac{1}{\mathbf{R}_x} = \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}\right) = -\frac{\partial^2 w}{\partial x^2}$$
$$\mathbf{K}_y = \frac{1}{\mathbf{R}_y} = \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y}\right) = -\frac{\partial^2 w}{\partial y^2} \qquad (59)$$

where $\mathbf{R}_x$ and $\mathbf{R}_y$ are main radia of curvatures. First (flexural) curvatures of the plate middle surface at the point $N(x,y,0)$ are:

$$\mathbf{K}_\xi = \frac{1}{\mathbf{R}_\xi} = \frac{\partial}{\partial \xi}\left(\frac{\partial w}{\partial \xi}\right) = -\frac{\partial^2 w}{\partial \xi^2} = -\frac{\partial}{\partial \xi}\left(\frac{\partial w}{\partial x}\frac{\partial x}{\partial \xi} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \xi}\right) =$$
$$= \mathbf{K}_x \cos^2\varphi + \mathbf{K}_y \sin^2\varphi - \mathbf{K}_{xy}\sin 2\varphi$$
$$\mathbf{K}_\eta = \frac{1}{\mathbf{R}_\eta} = \frac{\partial}{\partial \eta}\left(\frac{\partial w}{\partial \eta}\right) = -\frac{\partial^2 w}{\partial \eta^2} = -\frac{\partial}{\partial \eta}\left(\frac{\partial w}{\partial x}\frac{\partial x}{\partial \eta} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \eta}\right) = \qquad (60)$$
$$= \mathbf{K}_x \sin^2\varphi + \mathbf{K}_y \cos^2\varphi + \mathbf{K}_{xy}\sin 2\varphi$$

Semi sum of first (flexural) curvatures of the plate middle surface at the point $N(x,y,0)$ is middle curvature of the plate and is independent of orthogonal coordinates directions in this point. $\frac{\partial^2 w}{\partial x \partial y}$ is second curvature or torsion of the deformed plate middle surface, and we can express in the following form:

$$\mathfrak{T}_{\xi\eta} = \frac{\partial^2 w}{\partial \xi \partial \eta} = \frac{\partial}{\partial \xi}\left(\frac{\partial w}{\partial \eta}\right) = \frac{\partial}{\partial \xi}\left(\frac{\partial w}{\partial x}\cos\varphi + \frac{\partial w}{\partial y}\sin\varphi\right) = \frac{1}{2}(\mathbf{K}_x - \mathbf{K}_y)\sin 2\varphi + \mathfrak{T}_{xy}\cos 2\varphi \qquad (61)$$

Components of the tensor relative deformations at the plate point $N(x,y,z)$ are:

$$\varepsilon_x = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} = \frac{z}{\mathbf{R}_x}$$
$$\varepsilon_y = \frac{\partial u}{\partial y} = -z\frac{\partial^2 w}{\partial y^2} = \frac{z}{\mathbf{R}_y}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -2z\frac{\partial^2 w}{\partial x \partial y} = -2\frac{z}{\mathbf{R}_t} \tag{62}$$

4. 2* Constitutive relations of the stress and strain state of the plate stressed creep material.

Let's introduce the supposition that relations between stresses and strains, in the plate stressed and strained material with creeping properties, are the following relations:

$$\begin{aligned} \sigma_x &= \mathbf{E}_{0x}\varepsilon_x^x(t) + \mathbf{E}_{\alpha x}\mathfrak{D}_{\mathbf{t}}^{\alpha_x}\left[\varepsilon_x^x(t)\right] \\ \sigma_y &= \mathbf{E}_{0y}\varepsilon_y^y(t) + \mathbf{E}_{\alpha y}\mathfrak{D}_{\mathbf{t}}^{\alpha_y}\left[\varepsilon_y^y(t)\right] \\ \tau_{xy} &= \mathbf{G}_0\gamma_{xy}(t) + \mathbf{G}_\alpha\mathfrak{D}_t^{\alpha_{xy}}\left[\gamma_{xy}(t)\right] \end{aligned} \tag{63}$$

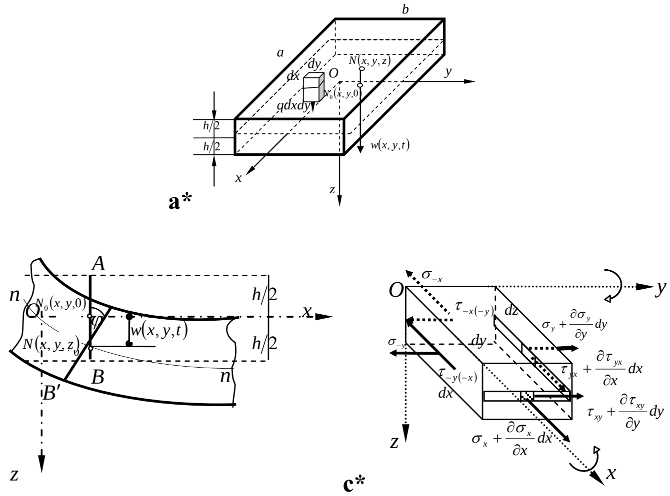


Figure 5. a* Thin plate with geometrical parameters. **b*** Plate cross section**. c*** Plate element with area $dxdy$ in the middle plate surface and with presentation of the unique height in cross section and corresponding stress tensor components in the plate cross sections.

where $\mathfrak{D}_{\mathbf{t}}^{\alpha_y}[*]$ differential operator with fractional order derivative defined by (2) and by material parameter α, which satisfy the following condition: $0 < \alpha < 1$. In the previous relations $\mathbf{E}_{0x}$, $\mathbf{E}_{0y}$, $\mathbf{E}_{\alpha x}$, $\mathbf{E}_{\alpha y}$ are the elasticity coefficients of loading plate material, momentous and prolongeous one in the corresponding axes directions

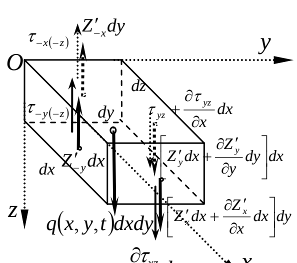
x and y; α_x, α_y and α_{xy} are corresponding coefficient of the creep of plate material for axial and shearing loads; and $\mathbf{G_0} = \dfrac{\mathbf{E}_0}{2(1+\mu)}$ and $\mathbf{G}_\alpha = \dfrac{\mathbf{E}_\alpha}{2(1+\mu)}$ are corresponding shear modulus.

On the basis of previous suppositions and relations we can write constitutive stress-strain relations. Now, into the previous equations – relation between stress components and strain components, we introduce the expression of strain tensor components expressed by transversal displacements $w(x,y,t)$ of the plate middle surface corresponding point $N(x,y,0)$ and coordinate z of the corresponding plate point $N(x,y,z)$. Than we obtain the following relations between stress components and transversal displacement $w(x,y,t)$. For homogeneous and isotropic material with parameters of material creep properties are equal $\alpha_x = \alpha_y = \alpha$; also, coefficients of rigidity of momentaneous and prolongeous one are: $\mathbf{E_{0x}} = \mathbf{E}_{\mathbf{0}y} = \mathbf{E_0}$ and $\mathbf{E}_{\alpha\mathbf{x}} = \mathbf{E}_{\alpha y} = \mathbf{E}_\alpha$ in all directions at corresponding point. For that case previous expressions are simplest and in the following form:

$$\sigma_x = -\frac{\mathbf{E}_0 z}{(1-\mu^2)}\left(\frac{\partial^2 w(x,y,t)}{\partial x^2} + \mu\frac{\partial^2 w(x,y,t)}{\partial y^2}\right) - \frac{\mathbf{E}_\alpha z}{(1-\mu^2)}\mathfrak{D}_t^\alpha\left[\frac{\partial^2 w(x,y,t)}{\partial x^2} + \mu\frac{\partial^2 w(x,y,t)}{\partial y^2}\right]$$

$$\sigma_y = -\frac{\mathbf{E}_0 z}{(1-\mu^2)}\left(\frac{\partial^2 w(x,y,t)}{\partial y^2} + \mu\frac{\partial^2 w(x,y,t)}{\partial x^2}\right) - \frac{\mathbf{E}_\alpha z}{(1-\mu^2)}\mathfrak{D}_t^\alpha\left[\frac{\partial^2 w(x,y,t)}{\partial y^2} + \mu\frac{\partial^2 w(x,y,t)}{\partial x^2}\right]$$

$$\tau_{xy} = -\frac{z\mathbf{E}_0}{(1+\mu)}\frac{\partial^2 w(x,y,t)}{\partial x\partial y} - \frac{z\mathbf{E}_\alpha}{(1+\mu)}\mathfrak{D}_t^\alpha\left[\frac{\partial^2 w(x,y,t)}{\partial x\partial y}\right] \tag{65}$$



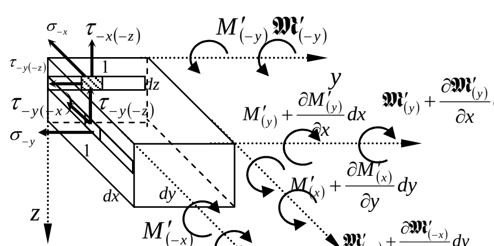


Figure 6. **a*** Plate element with area $dxdy$ in the middle plate surface and with presentation of the unique height in cross section and corresponding sheering stress components in z direction as well as the corresponding transversal forces.
b* Plate element with area $dxdy$ in the middle plate surface and with presentation of

the unique height in cross section and corresponding bending moments and moments of torsion.

When equilibrium conditions of the forces applied to the plate are satisfying, than it is necessary that Navier's equations of the equilibrium of every part of deformable body be satisfied. By using previous derived expressions of stress state tensor components σ_x, σ_y and τ_{xy} and introducing into Navier's equations of the equilibrium of every part of deformable body we obtain stress state tensor unknown components τ_{xz}, τ_{yz} and σ_z:

$$\tau_{xz}(x,y,z,t)=\tau_{zx}(x,y,z,t)=-\frac{\left(h^2-4z^2\right)}{8\left(1-\mu^2\right)}\left\{\mathbf{E}_0\frac{\partial}{\partial x}\Delta w(x,y,t)+\mathbf{E}_\alpha\mathfrak{D}_t^\alpha\left[\frac{\partial}{\partial x}\Delta w(x,y,t)\right]\right\}$$

$$\tau_{yz}(x,y,z,t)=\tau_{zy}(x,y,z,t)=-\frac{\left(h^2-4z^2\right)}{8\left(1-\mu^2\right)}\left\{\mathbf{E}_0\frac{\partial}{\partial y}\Delta w(x,y,t)+\mathbf{E}_\alpha\mathfrak{D}_t^\alpha\left[\frac{\partial}{\partial y}\Delta w(x,y,t)\right]\right\}$$

$$\sigma_z(x,y,z,t)=\frac{\left(3h^2z-4z^3-1\right)}{24\left(1-\mu^2\right)}\left\{\mathbf{E}_0\Delta\Delta w(x,y,t)+\mathbf{E}_\alpha\mathfrak{D}_t^\alpha\left[\Delta\Delta w(x,y,t)\right]\right\}+\rho g\left(\frac{h}{2}-z\right) \quad (65)$$

4. 3* Partial fractional order differential equation of the deformable plate middle surface.

By using boundary equilibrium condition, that the normal stress component σ_z for the upper plate surface is equal to the external normal surface loading $p(x,y,z,t)$, we can write the following:

$$\left\{\Delta\Delta w(x,y,t)+\kappa_\alpha\mathfrak{D}_t^\alpha\left[\Delta\Delta w(x,y,t)\right]\right\}=\frac{p(x,y,t)-\rho g h}{\mathfrak{D}_0} \quad (66)$$

where for cylindrical flexural rigidity $\mathfrak{D}_0$ and $\mathfrak{D}_\alpha$, momenteneous and prolongeneous one, of the loading processes to the plate material with creeping properties, as well as κ_α as ratio of these rigidities, are introduced in the following forms

$$\mathfrak{D}_0=\frac{\mathbf{E}_0}{12\left(1-\mu^2\right)},\qquad \mathfrak{D}_\alpha=\frac{\mathbf{E}_\alpha}{12\left(1-\mu^2\right)},\qquad \kappa_\alpha=\frac{\mathfrak{D}_\alpha}{\mathfrak{D}_0}=\frac{\mathbf{E}_\alpha}{\mathbf{E}_0} \quad .(67)$$

Relation between external plate surface excitation $p(x,y,t)$, and external volume excitation $\rho g h$ and transversal displacement $w(x,y,t)$ of the middle surface point $N(x,y)$ is in the form of partial fractional order differential equation (66).

Previous partial fractional order -differential equation is equation of the transversal displacement $w(x,y,t)$ of the middle surface point $N(x,y)$ loaded by external plate surface transversal excitation $p(x,y,t)$ and external volume excitation ρgh.
We conclude that for obtaining the last previous partial fractional order differential equation is equation of the transversal displacement $w(x,y,t)$ of the middle surface point $N(x,y)$ loaded by external plate surface transversal excitation $p(x,y,t)$ and external volume excitation ρgh, it is using an idea of Sophie Germain (1815) submitted by memoare to Paris academy of sciences, and corrected by Lagrange.

4. 4* Equation of quazi-statical equilibrium of a creep plate, and equation of transversal oscillations of a creep plate excited by external forces in the plate middle surface. Combined bending and stretching of rectangular plate.

In the previous consideration, the plate is assumed to bend with small deflection by lateral (transversal) loads only. If there are forces acting in the middle surface of the plate in addition to the lateral loads, the previous governed partial fractional differential equation must be modified to take into account the effects of these in-surface forces.

In general case of an elementary block with edges dx and dy, depth h, excited by external transversal surface forces $p(x,y,t)dxdy$, and forces $\mathbf{X}'$, $\mathbf{Y}'$ and $\mathbf{Y}_x' = \mathbf{X}_y'$ in the plate middle surface, is excited by surface forces caused by appearing of stresses, and their equivalent action as bending moments, moments of torsion and transversal forces. Also we can calculate the change of these surface forces, and moments caused by changing of the coordinates form x and y to the $x+dx$ and $y+dy$. Also we must calculate change of the normal direction of the cross section surface.

Aso, we must keep in mind that external forces components $\mathbf{X}'$ and $\mathbf{Y}'$ are applied in the plate middle surface in the cross section with coordinates x and y, and in cross sections with coordinates the $x+dx$ and $y+dy$ are: $\left(\mathbf{X}' + \frac{\partial \mathbf{X}'}{\partial x}dx\right)dy$, $\left(\mathbf{Y}' + \frac{\partial \mathbf{Y}'}{\partial y}dy\right)dx$, $\left(\mathbf{X}_y' + \frac{\partial \mathbf{X}_y'}{\partial y}dy\right)dx$ and $\left(\mathbf{Y}_x' + \frac{\partial \mathbf{Y}_x'}{\partial x}dx\right)dy$. We also, must keep in consideration that the parallel edges $\overline{AB}$ and $\overline{CD}$ are deformed on the distance dy, and that corresponding forces take with axis z the following angles β and $\beta + \frac{\partial \beta}{\partial y}dy$. Also, the parallel edges $\overline{AC}$ and $\overline{BD}$ are deformed on the distance

dx, and that corresponding forces take with axis z the following angles α and $\alpha + \frac{\partial \alpha}{\partial x} dx$.

By introducing all these elements, we can write equations of forces equilibrium acted to the elementary block with edges dx and dy, depth h in the following forms:

a* from the condition of the equilibrium into $0x$ direction;

b* from the condition of the equilibrium into $0y$ direction.

c* from the condition of the equilibrium into $0z$ direction, and we obtain final form of the transversal oscillations partial fractional order differential equation of the thin plate of the creep material:

$$\left\{\left(1+\kappa_\alpha \mathfrak{D}_t^\alpha\right)\left[\Delta\Delta w(x,y,t)\right]\right\} = \frac{p(x,y,t)}{\mathfrak{D}_0} + \frac{1}{\mathfrak{D}_0}\left(\mathbf{X}'\frac{\partial^2 w}{\partial x^2} + \mathbf{Y}'\frac{\partial^2 w}{\partial y^2} + 2\mathbf{X}_y'\frac{\partial^2 w}{\partial x \partial y}\right) - \frac{\rho h}{\mathfrak{D}_0}\frac{\partial^2 w(x,y,t)}{\partial t^2} \tag{68}$$

This equation is derived applying prepositions that proper weight of plate is neglected, and that depth of the plate is small.

4. 5* The basic partial fractional order differential equation and solution of the free plate creep oscillations.

By introducing notation $c_0^4 = \frac{\mathfrak{D}_0}{\rho h} = \frac{\mathbf{E}_0 h^3}{12\rho h\left(1-\mu^2\right)}$, and by using derived equations, we can obtain the partial fractional differential equation of the free plate transversal oscillations in the following form

$$\frac{\partial^2 w(x,y,t)}{\partial t^2} + c_0^4\left\{\left(1+\kappa_\alpha \mathfrak{D}_t^\alpha\right)\left[\Delta\Delta w(x,y,t)\right]\right\} = 0 \tag{69}$$

Solution of the previous fractional derivative-partial-differential equation can be looked for by using Bernoulli's method of particular integrals in the form of multiplication of two functions, from which the first $\mathbf{W}(x,y)$ depends only on space coordinates x and y, and the second is time function $T(t)$:

$$w(x,y,t) = \mathbf{W}(x,y)\mathrm{T}(t) \tag{70}$$

Assumed solution (70) is introduced in previous equation (69) and by introducing the notation of the constants:

$$\omega_0^2 = k^4 c_0^4 = k^4\frac{\mathfrak{D}_0}{\rho h} = k^4\frac{\mathbf{E}_0 h^2}{12\rho\left(1-\mu^2\right)}$$

$$\omega_\alpha^2 = \kappa_\alpha \omega_0^2 = k^4 \kappa_\alpha c_0^4 = k^4\frac{\mathfrak{D}_\alpha}{\rho h} = k^4\frac{\mathbf{E}_\alpha h^2}{12\rho\left(1-\mu^2\right)} \tag{71}$$

is easy to share previous equation on following two.

*first, a four order partial differential equation on unknown eigen function $\mathbf{W}(x,y)$ of space coordinates x and y in the form::

$$\Delta\Delta\mathbf{W}(x,y)-k^4\mathbf{W}(x,y)=0 \tag{72}$$

or in the form of two second order differential equations:

$$\Delta\mathbf{W}(x,y)\pm k^2\mathbf{W}(x,y)=0 \tag{73}$$

and * second, fractional-differential equation on unknown time-function $\mathbf{T}(t)$:

$$\ddot{\mathbf{T}}(t)+\omega_0^2\left(1+\kappa_\alpha\mathfrak{D}_t^\alpha\right)[\mathbf{T}(t)]=0 \tag{74}$$

4. 6* Space coordinates' eigen amplitude function and time function for the creep vibrations of the plate.

Let's consider that space coordinate proper function $\mathbf{W}(x,y)$ is in the form of $\mathbf{W}(x,y)=\mathbf{X}(x)\mathbf{Y}(y)$, and then we can write:

$$\mathbf{X}''(x)+\left(\pm n^2\pm k^2\right)\mathbf{X}(x)=0 \qquad \mathbf{Y}''(y)\mp n^2\mathbf{Y}(y)=0 \tag{75}$$

If plate is rectangular, and when we take into consideration a solution in Descartes' coordinates with free ends along contours then $\mathbf{X}(x):=\sin mx;\cos mx; shmx; Chmx$, where $m^2=\pm n^2\pm k^2$, and corresponding $\mathbf{Y}(y):=\sin ny;\cos ny; shny; Chny$.

If plate is in the circular form, then it is suitable to use polar-cylindrical coordinate system, and then the set of the partial differential equations in the space cylindrical-polar coordinates r , φ and z is:

$$\Delta\mathbf{W}(r,\varphi)\pm k^2\mathbf{W}(r,\varphi)=0\,,$$

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}\right)\mathbf{W}(r,\varphi)\pm k^2\mathbf{W}(r,\varphi)=0 \tag{76}$$

Solutions of the previous equations we write in the form $\mathbf{W}(r,\varphi)=\Phi(\varphi)\mathbf{R}(r)$ and after applying this solution we obtain the following system of ordinary differential equations:

$$\Phi''(\varphi)\pm n^2\Phi(\varphi)=0$$

$$\mathbf{R}''(r)+\frac{1}{r}\mathbf{R}'(r)+\left(\pm k^2\mp\frac{n^2}{r^2}\right)\mathbf{R}(r)=0 \tag{77}$$

Second equation from previous system has particular solutions in the form of Neuman's and Bessel's functions, but Neuman's functions for $r=0$ have infinite value, than particular solutions of this defined task (problem) are only Bessel's function first kind with real argument $\mathbf{J}_n(x)$ as well as with imaginary arguments $\mathbf{I}_n(x)$, where $x = kr$. Modified Bessel's function first kind with imaginary arguments $\mathbf{I}_n(x)$, with order n is in the following form:

$$\mathbf{I}_n(x) = (i)^{-n}\mathbf{J}_n(ix) = \frac{(-1)^n}{2\pi}\int_{-\pi}^{+\pi} e^{-x\cos t}\cos ntdt \tag{78}$$

If n is integer number, than this function satisfies the following differential equation:

$$\mathbf{I}_n''(ix) + \frac{1}{(ix)}\mathbf{I}_n'(ix) - \left(1 + \frac{n^2}{(ix)^2}\right)\mathbf{I}_n(ix) = 0 \tag{79}$$

By using previous considerations and study of equations (?) for their solutions in the polar coordinates for the circular plate, we can write the following expressions:

$$\Phi_n(\varphi) = C_n \sin(n\varphi + \varphi_{0n}) \tag{80}$$

$$\mathbf{R}_{nm}(r) = \mathbf{J}_n(k_{nm}r) + K_{nm}\mathbf{I}_n(k_{nm}r) \tag{81}$$

General solution for the transversal plate middle surface point displacement is in the following form:

$$w(r,\varphi,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\mathbf{J}_n(k_{nm}r) + K_{nm}\mathbf{I}_n(k_{nm}r)\right]\sin(n\varphi + \varphi_{0n})\mathbf{T}_{mn}(t) \tag{82}$$

where time functions $\mathbf{T}_{nm}(t)$ are in the form of series:

$$\begin{aligned}\mathbf{T}_{nm}(t) = \mathbf{T}_{0nm}\sum_{i=0}^{\infty}(-1)^i\omega_{\alpha nm}^{2i}t^{2i}\sum_{j=0}^{j=i}\binom{i}{j}\frac{\omega_{\alpha nm}^{2j}t^{-\alpha j}}{\omega_{0nm}^{2j}\Gamma(2i+1-\alpha j)} + \\ + \dot{\mathbf{T}}_{0nm}\sum_{i=0}^{\infty}(-1)^i\omega_{\alpha nm}^{2i}t^{2i-1}\sum_{j=0}^{j=i}\binom{i}{j}\frac{\omega_{\alpha nm}^{-2j}t^{-\alpha j}}{\omega_{0nm}^{2j}\Gamma(2i+2-\alpha j)}\end{aligned} \tag{83}$$

where we introduce the following notations:

$$\omega_{0nm} = k_{nm}^2 c_0^2 = k_{nm}^2\sqrt{\frac{\mathfrak{D}_0}{\rho h}} = k_{nm}^2\frac{h}{2}\sqrt{\frac{\mathbf{E}_0}{3\rho(1-\mu^2)}} \tag{84}$$

and

$$\omega_{\alpha nm} = \omega_{0nm}\sqrt{\kappa_\alpha} = k_{nm}^2 c_0^2\sqrt{\kappa_\alpha} = k_{nm}^2\sqrt{\frac{\mathfrak{D}_\alpha}{\rho h}} = k_{nm}^2\frac{h}{2}\sqrt{\frac{\mathbf{E}_\alpha}{3\rho(1-\mu^2)}}\,. \tag{85}$$

Time functions $\mathbf{T}_{nm}(t)$ are defined as a solution of the corresponding ordinary fractional differential equation defined by (11) or (74).

4. 7* Solution of the partial fractional differential equation of the free rectangular plate oscillations with the hinged edges on the plate contour.

Let's, now, study free oscillations of a rectangular plate with basic edges a and b, and with hinged edges on the middle surface plate contour – simply supported plate. Boundary conditions of this rectangular plate are that transversal displacements on the middle surface plate contour points equal zero, and also in same points the bending moments are equal to zero. By these basic conclusions, boundary conditions are expressed in the following forms:

for $x=0 \qquad w(0,y,t)=0$

$$\mathbf{M}'_{(x)}(0,y,t)=\mathbf{M}'_{yx}(0,y,t)=-\mathfrak{D}_0\left(1+\kappa_\alpha \mathfrak{D}_{\mathbf{t}}^{\alpha}\right)\left[\frac{\partial^2 w(0,y,t)}{\partial y^2}+\mu\frac{\partial^2 w(0,y,t)}{\partial x^2}\right]=0\,;$$

for $x=a \qquad w(a,y,t)=0$

$$\mathbf{M}'_{(x)}(a,y,t)=\mathbf{M}'_{yx}(a,y,t)=-\mathfrak{D}_0\left(1+\kappa_\alpha \mathfrak{D}_{\mathbf{t}}^{\alpha}\right)\left[\frac{\partial^2 w(a,y,t)}{\partial y^2}+\mu\frac{\partial^2 w(a,y,t)}{\partial x^2}\right]=0$$

for $y=0 \qquad w(x,0,t)=0$

$$\mathbf{M}'_{(y)}(x,0,t)=\mathbf{M}'_{xy}(x,0,t)=-\mathfrak{D}_0\left(1+\kappa_\alpha \mathfrak{D}_{\mathbf{t}}^{\alpha}\right)\left[\frac{\partial^2 w(x,0,t)}{\partial x^2}+\mu\frac{\partial^2 w(x,0,t)}{\partial y^2}\right]=0 \qquad (86)$$

for $y=b \qquad w(x,b,t)=0$

$$\mathbf{M}'_{(y)}(x,b,t)=\mathbf{M}'_{xy}(x,b,t)=-\mathfrak{D}_0\left(1+\kappa_\alpha \mathfrak{D}_{\mathbf{t}}^{\alpha}\right)\left[\frac{\partial^2 w(x,b,t)}{\partial x^2}+\mu\frac{\partial^2 w(x,b,t)}{\partial y^2}\right]=0$$

Partial differential equation (72) with accomplished boundary conditions (86) are satisfied by the following solution:

$$\mathbf{W}_{mn}(x,y)=C_{mm}\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y \qquad (87)$$

where are:

$$k_{mn}^2=\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]$$

$$\omega_{0nm}=\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]c_0^2=\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]\frac{h}{2}\sqrt{\frac{\mathbf{E}_0}{3\rho\left(1-\mu^2\right)}} \qquad (88)$$

Solution of the transversal middle surface plate point displacements for free vibrations of the rectangular plate with creep properties of material, hinged on the contour is in the form:

$$w(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\mathbf{T}_{mn}(t)\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y \tag{89}$$

where time function $\mathbf{T}_{nm}(t)$ in the form (83). Than finally we obtain:

$$w(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[\mathbf{T}_{0nm}\sum_{i=0}^{\infty}(-1)^i\omega_{\alpha nm}^{2i}t^{2i}\sum_{j=0}^{j=i}\binom{i}{j}\frac{\omega_{\alpha nm}^{2j}t^{-\alpha j}}{\omega_{0nm}^{2j}\Gamma(2i+1-\alpha j)}\right]\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y+$$
$$+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[\dot{\mathbf{T}}_{0nm}\sum_{i=0}^{\infty}(-1)^i\omega_{\alpha nm}^{2i}t^{2i-1}\sum_{j=0}^{j=i}\binom{i}{j}\frac{\omega_{\alpha nm}^{-2j}t^{-\alpha j}}{\omega_{0nm}^{2j}\Gamma(2i+2-\alpha j)}\right]\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y \tag{90}$$

where $\mathbf{T}_{0mn}$ and $\dot{\mathbf{T}}_{0mn}$ are integral constants defined by initial conditions.

4. 8* . Numerical experiments and results.

By using the expression obtained for time function $\mathbf{T}_{nm}(t)$ with corresponding particular solutions, we made numerical experiment for characteristic cases and ratios of plate parameters, coefficient α of creeping material and results are presented in the following Figures.

In Figure 3. numerical simulations and graphical presentation of the solution (85) of the fractional-differential equation (64) of the system are presented, in analogy with corresponding for transversal vibrations of the beam. Time functions $T(t,\alpha)$ surfaces for different plate transversal vibrations kinetic and creep material parameters in the space $(T(t,\alpha),t,\alpha)$ for interval $0\le\alpha\le1$ are visible: in **a*** for $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right)=1$; in **b*** for $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right)=\frac{1}{4}$; in **c*** for $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right)=\frac{1}{3}$; in **d*** for $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right)=3$.

In Figure 4. the time functions $T(t,\alpha)$ surfaces and curves families for the different plate transversal vibrations kinetic and discrete values of the creep material parameters $0\le\alpha\le1$ are presented, in analogy with corresponding for transversal vibrations of the beam. In Figures **a*** and **c*** for $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right)=1$; in Figures b* and **d*** for

$\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right)=\frac{1}{4}$; in Figure **e*** for $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right)=\frac{1}{3}$; and in Figure **f*** for $\left(\frac{\omega_{\alpha x}}{\omega_{0x}}\right)=3$.

In this paper, new theory of the deformation and oscillations of thin creeping material plate for small deformation is presented. Creeping properties of a material plate are presented by using constitutive relations in the fractional differential form with terms by fractional order derivative with respect to time. By using presented assumptions and pointed out theory, the partial fractional differential equations of the quazi-static equilibrium of plate, and transversal oscillations are derived, and solved for different deformation as well as oscillations creeping state are obtained, for different boundary plate conditions.

Also, the expressions of the stress tensor components distributions in the plate of the creeping properties material are derived. The expressions of the bending and twisting moments and transversal force are derived.

From the obtained analytical and numerical results for free transversal creep vibrations of a fractional derivative order hereditary homogeneous thin plate, it can be seen that a fractional derivative order hereditary properties in all cases are convenient for changing time function depending on material creep parameters, and that fundamental eigen-function depending on space coordinates is dependent only on boundary conditions and geometrical properties of plate.

5. DOUBLE PLATE SYSTEM

5. 1* Theoretical problem formulation and governing equations.

Let us suppose that both plates in double plate systems satisfy same conditions as the plate considered in the previous paragraph ИII and listed in the beginning of the sub-paragraph 4.1*.

Now, let us consider two isotropic, creeping, thin plates, with thickness h_i, $i=1,2$, modulus of elasticity E_i and $E_{i\partial}$, Poisson's ratio μ_i and shear modulus G_i, plate mass distribution ρ_i. The plates are of constant thickness in the z-direction (see Fig. 7). The contours of the both plates are parallel and same type of the boundary conditions. Plate is interconnected by a creeping layer with the fractional order derivative constitutive relation type with constant surface stiffness. This creep-layer connected double plate system is of composite structure type, or sandwich plates, or layered plates.

The origins of the two coordinate systems are two corresponding sets at the corresponding centres of the no deformed plates middle surfaces as shown in Fig. 7. and with parallel corresponding axes. The both plates may be subjected to either transversal distributed external loads $q_i(x,y,t)$, $i=1,2$ along corresponding plates external boundary (contour) surfaces parallel to the corresponding plate middle surface. The problem at hand is to determine solutions.

The use of Love-Kirchhoff approximation make classical plate theory essentially a two dimensional phenomenon, in which the normal and transverse forces and bending and twisting moments on plate cross sections (see Ref. [88, 89] books by Rašković (1965) and (1985)) can be found in terms of displacement $w_i(x,y,t)$, $i=1,2$ of the middle surface points, which is assumed to be a function of two coordinates, x and y and time t ., as it is considered for one plate in the previous chapter 4.

The plates are assumed to be with same contour forms and boundary conditions.

Let us suppose that the plate middle surfaces are planes in an underformed state system. If the plates transverse deflections $w_i(x,y,t)$, $i=1,2$ are small (in the sense, as has been discussed in Refs. [88, 89] books by Rašković (1965) and (1985), small compared to the plates thickness, h_i, $i=1,2$,) and that plates vibrations occur only in the vertical direction. Let us denote with $\mathfrak{D}_i=\dfrac{\mathbf{E}_i h^3}{12(1-\mu^2)}$, $\mathfrak{D}_{i\alpha}=\dfrac{\mathbf{E}_{i\alpha} h^3}{12(1-\mu^2)}$, $i=1,2$ the corresponding bending cylindrical rigidity of creep plates, analogous as in the previous paragraph VII. For homogeneous and isotropic plates material the parameters of material creep properties are equal in all directions, i.e. $\alpha_x=\alpha_y=\alpha$; also, coefficients of rigidity of momentaneous and prolongeous one are: $\mathbf{E_{0x}}=\mathbf{E_{0}}_y=\mathbf{E_0}$ and $\mathbf{E}_{\alpha\mathbf{x}}=\mathbf{E}_{\alpha y}=\mathbf{E}_\alpha$ in all directions at corresponding point.. The coefficients of rigidity of momentaneous and prologues one for creep layer are c and c_α, and the parameter of layer material creep properties is $0\le\alpha\le 1$.

Now, by using results from References [38, 57] by (Stevanović) Hedrih (2003), the relation between stress components and strain components are expressed by transversal displacements $w(x,y,t)$ of the plate middle surface corresponding point $N(x,y,0)$ and coordinate z of the corresponding plate point $N(x,y,z)$. Than we can write the following relations between stress components and transversal displacement $w(x,y,t)$ in the form (64) and (65).

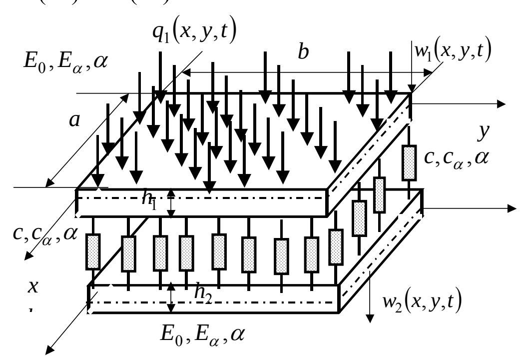


Figure 7. A creeping connected double plate system

By using results of (Stevanović) Hedrih (2003, 2004, 2004a), as well as results from previous paragraph III. the governing partial fractional order differential equations of the creep connected double plate system dynamics are formulated in terms of two unknowns: the transversal displacement $w_i(x,y,t)$, $i=1,2$ in direction of the axis z, of the upper plate middle surface and of the lower plate middle surface (Fig. 7). The system of these two coupled partial fractional order differential equations are derived by using d'Alembert's principle of dynamical equilibrium. These partial differential equations of the creeping connected double plate system are in the following forms:

$$\frac{\partial^2 w_1(x,y,t)}{\partial t^2} + c_{(1)}^4\left\{\left(1+\kappa_\alpha \mathfrak{D}_t^\alpha\right)\left[\Delta\Delta w_{(1)}(x,y,t)\right]\right\} - \\ - a_{(1)}^2\left\{\left(1+\kappa_\alpha^c \mathfrak{D}_t^\alpha\right)\left[w_2(x,y,t)-w_1(x,y,t)\right]\right\} = \tilde{q}_1(x,y,t)$$

$$\frac{\partial^2 w_2(x,y,t)}{\partial t^2} + c_{(2)}^4\left\{\left(1+\kappa_\alpha \mathfrak{D}_t^\alpha\right)\left[\Delta\Delta w_2(x,y,t)\right]\right\} + \\ + a_{(2)}^2\left\{\left(1+\kappa_\alpha^c \mathfrak{D}_t^\alpha\right)\left[w_2(x,y,t)-w_1(x,y,t)\right]\right\} = -\tilde{q}_2(x,y,t) \tag{91}$$

where: $c_{(i)}^4 = \frac{\mathfrak{D}_i}{\rho_i h_i} = \frac{\mathbf{E}_0 h_i^3}{12\rho_i h_i(1-\mu^2)}$, $\kappa_\alpha = \frac{\mathbf{E}_\alpha}{\mathbf{E}_0}$, $\kappa_\alpha^c = \frac{c_\alpha}{c}$, $a_{(i)}^2 = \frac{c}{\rho_i h_i}$,

$\tilde{q}_i(x,y,t) = \frac{q_i(x,y,t)}{\rho_i h_i}$ $i=1,2$.

5. 2* Solution of the governing equations.

Solution of the previous partial fractional order differential equations (91) can be presumed for the Bernoulli's method of particular integrals in the form of multiplication of two functions (see books [78, 79] by Rašković (1965) and (1985), and previous paragraph VII), the first of which $\mathbf{W}_{(i)}(x,y)$, $i=1,2$ depends only on space coordinates x and y, and the second is a time function $T_{(i)}(t)$, $i=1,2$:

$$w_{(i)}(x,y,t) = \mathbf{W}_{(i)}(x,y)\mathrm{T}_{(i)}(t)\ ,\ i=1,2 \tag{92}$$

For the beginning, the assumed solution (92) is introduced in previous system of partial fractional order differential equations (91) for the case of free vibrations, when external surface excitation $q_i(x,y,t)=0$, $i=1,2$ equal zero and we obtain the following:

$$\ddot{T}_{(1)}(t)+c_{(1)}^4\left\{\left(1+\kappa_\alpha \mathfrak{D}_t^\alpha\right)\left[T_{(1)}(t)\right]\right\}\frac{\Delta\Delta \mathbf{W}_{(1)}(x,y)}{\mathbf{W}_{(1)}(x,y)}-$$
$$-a_{(1)}^2\left\{\left(1+\kappa_\alpha^c \mathfrak{D}_t^\alpha\right)\left[\frac{\mathbf{W}_{(2)}(x,y)}{\mathbf{W}_{(1)}(x,y)}T_{(2)}(t)-T_{(1)}(t)\right]\right\}=0$$
$$\ddot{T}_{(2)(t)}+c_{(2)}^4\left\{\left(1+\kappa_\alpha \mathfrak{D}_t^\alpha\right)\left[T_{(2)(t)}\right]\right\}\frac{\Delta\Delta \mathbf{W}_{(1)}(x,y)}{\mathbf{W}_{(1)}(x,y)}+$$
$$+a_{(2)}^2\left\{\left(1+\kappa_\alpha^c \mathfrak{D}_t^\alpha\right)\left[\left[\frac{\mathbf{W}_{(2)}(x,y)}{\mathbf{W}_{(1)}(x,y)}T_{(2)}(t)-T_{(1)}(t)\right]\right]\right\}=0 \quad (93)$$

After analysis of the previously obtained system of equations, we can write the corresponding systems of the basic decoupled system equations in the following forms:

$$\ddot{\mathrm{T}}_{(1)}(t)+\omega_{(1)}^2\left(1+\widetilde{\kappa}_{\alpha(1)}\mathfrak{D}_t^\alpha\right)\mathrm{T}_{(1)}(t)=0$$
$$\ddot{\mathrm{T}}_{(2)}(t)+\omega_{(2)}^2\left(1+\widetilde{\kappa}_{\alpha(21)}\mathfrak{D}_t^\alpha\right)\mathrm{T}_{(2)}(t)=0 \quad (94)$$

where $\omega_{(i)}^2=k_{(i)}^4c_{(i)}^4+a_{(i)}^2$; $\widetilde{\kappa}_{\alpha(i)}=\dfrac{k_{(i)}^4c_{(1)}^2\kappa_\alpha+a_{(i)}^2\kappa_\alpha^c}{k_{(i)}^2c_{(1)}^2+a_{(1)}^2}$.

a* If plates are rectangular, then we can take into consideration a system in Descartes' coordinates:

$$\Delta\Delta\,\mathrm{W}_{(1)}(x,y)-k_{(1)}^4\,\mathrm{W}_{(1)}(x,y)=0\,,\ \mathbf{W}_{(1)}(x,y)=\mathbf{W}_{(2)}(x,y)\,, \quad (95)$$

b* If plates are in the circular form, then it is suitable to use polar-cylindrical coordinate system, and then the set of the partial differential equations $\mathbf{W}_{(i)}(r,\varphi)$, $i=1,2$ in the space of cylindrical-polar coordinates r, φ and z is:

$$\Delta\Delta\,\mathrm{W}_{(1)}(r,\varphi)-k_{(1)}^4\,\mathrm{W}_{(1)}(r,\varphi)=0\,,\ \mathbf{W}_{(1)}(r,\varphi)=\mathbf{W}_{(2)}(r,\varphi) \quad (96)$$

$$\omega_{(1)}^2=k_{(1)}^4c_{(1)}^4+a_{(1)}^2\ ,\ \omega_{(2)}^2=k_{(2)}^4c_{(2)}^4+a_{(2)}^2$$

where there are introduced the notation of the system own parameters.

$$\omega_{(i)}^2=k^4c_{(i)}^4+a_{(i)}^2 \qquad \text{and} \qquad \omega_{(i)\alpha}^2=\widetilde{\kappa}_{(i)\alpha}\omega_{(i)}^2=k^4\kappa_\alpha c_{(i)}^4+a_{(i)}^2\kappa_{(i)\alpha}^c \quad (97)$$

The solutions of partial differential equations (95) and (96) are known from classical literature (see books by Rašković (1965), and in the previous paragraph VII.6* and VII. 7*), and are eigen amplitude plate functions defined shape of amplitude plate displacements for corresponding eigen circular free vibrations. These eigen amplitude functions satisfy boundary conditions, and also orthogonality conditions. For the corresponding boundary conditions, the space coordinate own amplitude functions we denote with $\mathbf{W}_{(i)nm}(x,y)$, $i=1,2$, $n,m=1,2,3,4,....\infty$ and we can write the following conditions of orthogonality:

$$\int_0^a\int_0^b \mathrm{W}_{(i)mn}(x,y)\mathrm{W}_{(i)sr}(x,y)dxdy=\begin{cases}0 & nm\neq sr\\ v_{mnnm} & nm=sr\end{cases}, \tag{98}$$

$i=1,2$, $n,m=1,2,3,4,....\infty$, $s,r=1,2,3,4,....\infty$

which is easily derived by using a system of equations (95) or (96) for different pairs nm and sr. Also, the time functions $\mathbf{T}_{(i)nm}(t)$, $i=1,2$, $n,m=1,2,3,4,....\infty$ are expressed in the form of series similar as in the previous paragraph expressed by formula (67). Then, we can write:

$$\mathbf{T}_{(i)nm}(t)=\mathbf{T}_{0(i)nm}\sum_{i=0}^{\infty}(-1)^i\omega_{(i)\alpha nm}^{2i}t^{2i}\sum_{j=0}^{j=i}\binom{i}{j}\frac{\omega_{(ii)\alpha nm}^{2j}t^{-\alpha j}}{\omega_{(i)nm}^{2j}\Gamma(2i+1-\alpha j)}+$$
$$+\dot{\mathbf{T}}_{0(i)nm}\sum_{i=0}^{\infty}(-1)^i\omega_{(i)\alpha nm}^{2i}t^{2i-1}\sum_{j=0}^{j=i}\binom{i}{j}\frac{\omega_{(i)\alpha nm}^{-2j}t^{-\alpha j}}{\omega_{(i)nm}^{2j}\Gamma(2i+2-\alpha j)} \tag{99}$$

$i=1,2$, $n,m=1,2,3,4,....\infty$

where the following notations:

$\omega_{(i)nm}^2=k_{nm}^4c_{(i)}^4+a_{(i)}^2$ *and*

$$\omega_{(i)\alpha nm}^2=\tilde{\kappa}_{(i)\alpha nm}\omega_{(i)nm}^2=k_{nm}^4\kappa_\alpha c_{(i)}^4+a_{(i)}^2\kappa_{(i)\alpha}^c,\ i=1,2,\ n,m=1,2,3,4,....\infty \tag{100}$$

are introduced and k_{nm} are own characteristic number defined as a series of roots of characteristic equation obtained from corresponding plate boundary conditions, as it is known.

The solutions of the governing system of corresponding coupled partial fractional order differential equations (94) for free double plates oscillations, we take in the eigen amplitude function $\mathbf{W}_{(i)nm}(x,y)$, $i=1,2$, $n,m=1,2,3,4,....\infty$ expansion, from solution of the previous basic problem with decoupled equations, and with time coefficients in the form of unknown time functions $T_{(i)nm}(t)$, $i=1,2$, $n,m=1,2,3,4,....\infty$ describing their time evolution.

$$w_1(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\mathrm{W}_{(1)nm}(x,y)\mathrm{T}_{(1)nm}(t)$$

$$w_2(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\mathrm{W}_{(2)nm}(x,y)\mathrm{T}_{(2)nm}(t) \tag{101}$$

where the eigen amplitude functions $\mathbf{W}_{(i)nm}(x,y)$, $i=1,2$, $n,m=1,2,3,4,....\infty$ are same as in the case with decoupled plates problem, previous considered in the paragraph VII. Then after introducing the (111) and (112) into the governing system of coupled partial fractional order differential equations for free and also for forced double plates oscillations (91) and by multiplying first and second equation with $\mathrm{W}_{(i)sr}(x,y)dxdy$ and after integrating along all surfaces of the plate middle surface area and taking into account orthogonality conditions (98) and corresponding equal boundary conditions of the plates we obtain the mn-family of systems containing only two coupled ordinary fractional order differential equations for determination of the unknown time functions $T_{(i)nm}(t)$, $i=1,2$, $n,m=1,2,3,4,....\infty$ in the following form:

$$\begin{aligned}
&\ddot{\mathrm{T}}_{(1)nm}(t)+\omega_{(1)nm}^{2}\left(1+\widetilde{\kappa}_{\alpha(1)nm}\mathfrak{D}_{\mathbf{t}}^{\alpha}\right)\mathrm{T}_{(1)nm}(t)-\\
&\qquad-\left(a_{(1)}^{2}+a_{(1)\alpha\,nm}^{2}\mathfrak{D}_{\mathbf{t}}^{\alpha}\right)\mathrm{T}_{(2)nm}(t)=f_{(1)nm}(t)\\
&\ddot{\mathrm{T}}_{(2)nm}(t)+\omega_{(2)nm}^{2}\left(1+\widetilde{\kappa}_{\alpha(2)nm}\mathfrak{D}_{\mathbf{t}}^{\alpha}\right)\mathrm{T}_{(2nm)}(t)-\\
&\qquad-\left(a_{(2)}^{2}+a_{(2)\alpha\,nm}^{2}\mathfrak{D}_{\mathbf{t}}^{\alpha}\right)\mathrm{T}_{(1)nm}(t)=-f_{(2)nm}(t)\\
&\qquad n,m=1,2,3,4,....\infty
\end{aligned}\tag{113}$$

where time known function $f_{(1)nm}(t)$ and $f_{(2)nm}(t)$ are defined by following expressions:

$$f_{(1)nm}(t)=\frac{\int_0^a\int_0^b\widetilde{q}_1(x,y,t)\mathrm{W}_{(1)nm}(x,y)dxdy}{\int_0^a\int_0^b\left[\mathrm{W}_{(1)nm}(x,y)\right]^2dxdy}$$

$$f_{(2)nm}(t)=\frac{\int_0^a\int_0^b\widetilde{q}_2(x,y,t)\mathrm{W}_{(2)nm}(x,y)dxdy}{\int_0^a\int_0^b\left[\mathrm{W}_{(2)nm}(x,y)\right]^2dxdy}\tag{114}$$

The system of coupled fractional order differential equations (113) on unknown time-functions $T_{(i)nm}(t)$, $i=1,2$, $n,m=1,2,3,4,....\infty$, can be solved applying Laplace's transforms. Upon that fact Laplace transform of solutions is in forms:

$$\mathfrak{L}\left[T_{(1)nm}(t)\right]=\frac{1}{\Delta_{nm}(p)}\left[pT_{(1)nm}(0)+\dot{T}_{(1)nm}(0)+\mathfrak{L}\left[f_{(1)nm}(t)\right]\right]\cdot\left\{p^2+\omega_{(2)nm}^2\left[1+\frac{\omega_{(2)\alpha\,nm}^2}{\omega_{(2)nm}^2}\mathbf{R}(p)\right]\right\}+$$

$$+\frac{1}{\Delta_{nm}(p)}\left[pT_{(2)nm}(0)+\dot{T}_{(2)nm}(0)-\mathfrak{L}\left[f_{(2)nm}(t)\right]\right]\cdot\left\{a_{(1)}^{2}\left[1+\frac{a_{(1)\alpha}^{2}}{\omega_{(1)}^{2}}\mathbf{R}(p)\right]\right\} \quad (115)$$

$$\mathfrak{L}\left[T_{(2)nm}(t)\right]=\frac{1}{\Delta_{nm}(p)}\left[pT_{(2)nm}(0)+\dot{T}_{(2)nm}(0)-\mathfrak{L}\left[f_{(2)nm}(t)\right]\right]\cdot\left\{p^{2}+\omega_{(1)nm}^{2}\left[1+\frac{\omega_{(1)\alpha\, nm}^{2}}{\omega_{(1)nm}^{2}}\mathbf{R}(p)\right]\right\}+$$

$$+\frac{1}{\Delta_{nm}(p)}\left[pT_{(1)nm}(0)+\dot{T}_{(1)nm}(0)+\mathfrak{L}\left[f_{(1nm)}(t)\right]\right]\cdot\left\{a_{(2)}^{2}\left[1+\frac{a_{(2)\alpha}^{2}}{\omega_{(2)}^{2}}\mathbf{R}(p)\right]\right\} \quad (116)$$

where $\Delta_{nm}(p)$ is determinant of the system of equations obtained by Laplace's transform of the system of equations (13):

$$\Delta_{nm}(p)=\begin{vmatrix} p^{2}+\omega_{(1)nm}^{2}\left[1+\frac{\omega_{(1)\alpha\, nm}^{2}}{\omega_{(1)nm}^{2}}\mathbf{R}(p)\right] & -a_{(1)}^{2}\left[1+\frac{a_{(1)\alpha}^{2}}{\omega_{(1)}^{2}}\mathbf{R}(p)\right] \\ -a_{(2)}^{2}\left[1+\frac{a_{(2)\alpha}^{2}}{\omega_{(2)}^{2}}\mathbf{R}(p)\right] & p^{2}+\omega_{(2)nm}^{2}\left[1+\frac{\omega_{(2)\alpha\, nm}^{2}}{\omega_{(2)nm}^{2}}\mathbf{R}(p)\right] \end{vmatrix}\neq 0 \quad (117)$$

$\mathfrak{L}\left[\mathfrak{D}_{t}^{\alpha}\left[T_{(i)nm}(t)\right]\right]=\mathbf{R}(p)\mathfrak{L}\left[\mathrm{T}_{(i)nm}(\mathrm{t})\right]$ is Laplace transform of a fractional derivative $\frac{d^{\alpha}T_{(i)nm}(t)}{dt^{\alpha}}$ for $0\leq\alpha\leq1$. For creep rheological material those Laplace transforms has the form:

$$\mathfrak{L}\left[\mathfrak{D}_{t}^{\alpha}\left[T_{(i)nm}(t)\right]\right]=\mathbf{R}(p)\mathfrak{L}\left[T_{(i)nm}(t)\right]-\frac{d^{\alpha-1}}{dt^{\alpha-1}}T_{(i)nm}(0)= = p^{\alpha}\mathfrak{L}\left[T_{(i)nm}(t)\right]-\frac{d^{\alpha-1}}{dt^{\alpha-1}}T_{(i)nm}(0) \quad (118)$$

where the initial value are:

$$\left.\frac{d^{\alpha-1}T_{(i)nm}(t)}{dt^{\alpha-1}}\right|_{t=0}=0 \quad (118a)$$

so, in that case Laplace transforms of time-functions are given by following expressions:

$$\mathfrak{L}\left[T_{(1)nm}(t)\right]=\frac{1}{\Delta_{nm}(p)}p^{2}\left[pT_{(1)nm}(0)+\dot{T}_{(1)nm}(0)+\mathfrak{L}\left[f_{(1)nm}(t)\right]\right]\cdot\left\{1+\frac{\omega_{(2)\alpha nm}^{2}}{p^{2}}\left[p^{\alpha}+\frac{\omega_{(2)nm}^{2}}{\omega_{(2)\alpha\, nm}^{2}}\right]\right\}+$$

$$+\frac{1}{\Delta_{nm}(p)}\left[pT_{(2)nm}(0)+\dot{T}_{(2)nm}(0)-\mathfrak{L}\left[f_{(2)nm}(t)\right]\right]\cdot\left\{a_{(1)\alpha}^{2}\left[p^{\alpha}+\frac{a_{(1)}^{2}}{\omega_{(1)\alpha}^{2}}\right]\right\} \tag{119}$$

$$\mathfrak{L}\left[T_{(2)nm}(t)\right]=\frac{1}{\Delta_{nm}(p)}p^{2}\left[pT_{(2)nm}(0)+\dot{T}_{(2)nm}(0)-\mathfrak{L}\left[f_{(2)nm}(t)\right]\right]\cdot\left\{1+\frac{\omega_{(1)\alpha nm}^{2}}{p^{2}}\left[p^{\alpha}+\frac{\omega_{(1)nm}^{2}}{\omega_{(1)\alpha nm}^{2}}\right]\right\}+$$
$$+\frac{1}{\Delta_{nm}(p)}\left[pT_{(1)nm}(0)+\dot{T}_{(1)nm}(0)+\mathfrak{L}\left[f_{(1nm)}(t)\right]\right]\cdot\left\{a_{(2)\alpha}^{2}\left[p^{\alpha}+\frac{a_{(2)}^{2}}{\omega_{(2)\alpha}^{2}}\right]\right\} \tag{119a}$$

where $\Delta_{nm}(p)$ is determinant of the system equations obtained by Laplace's transform of the system equations (13):

$$\Delta_{nm}(p)=\begin{vmatrix} p^{2}\left\{1+\frac{\omega_{(1)\alpha nm}^{2}}{p^{2}}\left[p^{\alpha}+\frac{\omega_{(1)nm}^{2}}{\omega_{(1)\alpha nm}^{2}}\right]\right\} & -a_{(1)\alpha}^{2}\left[p^{\alpha}+\frac{a_{(1)}^{2}}{\omega_{(1)\alpha}^{2}}\right] \\ -a_{(2)\alpha}^{2}\left[p^{\alpha}+\frac{a_{(2)}^{2}}{\omega_{(2)\alpha}^{2}}\right] & p^{2}\left\{1+\frac{\omega_{(2)\alpha nm}^{2}}{p^{2}}\left[p^{\alpha}+\frac{\omega_{(2)nm}^{2}}{\omega_{(2)\alpha nm}^{2}}\right]\right\} \end{vmatrix}\neq 0 \tag{120}$$

$$\Delta_{nm}(p)=p^{4}\left\{1+\frac{\omega_{(1)\alpha nm}^{2}}{p^{2}}\left[p^{\alpha}+\frac{\omega_{(1)nm}^{2}}{\omega_{(1)\alpha nm}^{2}}\right]\right\}\cdot\left\{1+\frac{\omega_{(2)\alpha nm}^{2}}{p^{2}}\left[p^{\alpha}+\frac{\omega_{(2)nm}^{2}}{\omega_{(2)\alpha nm}^{2}}\right]\right\}-$$
$$-a_{(1)\alpha}^{2}a_{(2)\alpha}^{2}\left[p^{\alpha}+\frac{a_{(1)}^{2}}{\omega_{(1)\alpha}^{2}}\right]\left[p^{\alpha}+\frac{a_{(2)}^{2}}{\omega_{(2)\alpha}^{2}}\right] \tag{120a}$$

5. 3* Vibration modes in dynamics of the homogeneous double plate fractional order systems.

We can consider these ordinary fractional order differential equations (113) by new coordinates:

$$\xi_{(1)nm}(t)=\frac{1}{2}\left\{T_{(1)nm}(t)+T_{(2)nm}(t)\right\}$$
$$\xi_{(2)nm}(t)=\frac{1}{2}\left\{T_{(1)nm}(t)-T_{(2)nm}(t)\right\} \tag{121}$$

in the following form

$$\ddot{\xi}_{(1)nm}(t)+\omega_{(1)nm}^{2}\left(1+\widetilde{\kappa}_{\alpha(1)nm}\mathfrak{D}_{t}^{\alpha}\right)\xi_{(1)nm}(t)-$$
$$-\left(a_{(1)}^{2}+a_{(1)\alpha nm}^{2}\mathfrak{D}_{t}^{\alpha}\right)\xi_{(1)nm}(t)=\frac{1}{2}\left[f_{(1)nm}(t)+f_{(2)nm}(t)\right]$$

$$\ddot{\xi}_{(2)nm}(t)+\omega_{(1)nm}^{2}\left(1+\widetilde{\kappa}_{\alpha(1)nm}\mathfrak{D}_{t}^{\alpha}\right)\xi_{(2)nm}(t)+ \\ +\left(a_{(1)}^{2}+a_{(1)\alpha\,nm}^{2}\mathfrak{D}_{t}^{\alpha}\right)\xi_{(2)nm}(t)=\frac{1}{2}\left[f_{(1)nm}(t)-f_{(2)nm}(t)\right] \tag{122}$$

For the corresponding linear autonomous eigen frequencies are: $\omega_{1,nm}^{2}=\omega_{(1)nm}^{2}-a_{(1)}^{2}$ and $\omega_{1,nm}^{2}=\omega_{(1)nm}^{2}+a_{(1)}^{2}$, and corresponding solutions can be expressed by main linear system coordinates in the form: $T_{(1)nm}(t)=\xi_{(1)nm}(t)+\xi_{(2)nm}(t)$ and $T_{(2)nm}(t)=\xi_{(1)nm}(t)-\xi_{(2)nm}(t)$, where with $\xi_{(1)nm}(t)$ and $\xi_{(2)nm}(t)$, se denote corresponding oscillatory modes in the linear system as $\xi_{(1)nm}(t)=C_{(1)nm}\cos\left(\omega_{1.nm}t+\alpha_{(1)nm}\right)$ and $\xi_{(2)nm}(t)=C_{(2)nm}\cos\left(\omega_{2,nm}t+\alpha_{(2)nm}\right)$,. For the linear autonomous system case $C_{(s)nm}$ and $\alpha_{(s)nm}$ are constants depending of initial conditions.

By introducing following analogous notations: $\omega_{1\alpha,nm}^{2}=\omega_{(1)\alpha nm}^{2}-a_{(1)\alpha}^{2}$ and $\omega_{2\alpha,nm}^{2}=\omega_{(1)\alpha nm}^{2}+a_{(1)\alpha}^{2}$ to the corresponding linear eigen frequencies system of ordinary fractional order differential equations, (122) can be transform in the following form:

$$\ddot{\xi}_{(1)nm}(t)+\left(\omega_{1,nm}^{2}+\omega_{1\alpha,nm}^{2}\mathfrak{D}_{t}^{\alpha}\right)\left[\xi_{(1)nm}(t)\right]=\frac{1}{2}\left[f_{(1)nm}(t)-f_{(s)nm}(t)\right]$$

$$\ddot{\xi}_{(2)nm}(t)+\left(\omega_{2,nm}^{2}+\omega_{2\alpha,nm}^{2}\mathfrak{D}_{t}^{\alpha}\right)\left[\xi_{(2)nm}(t)\right]=\frac{1}{2}\left[f_{(1)nm}(t)+f_{(s)nm}(t)\right] \tag{122a}$$

where $\xi_{(1)nm}(t)$ and $\xi_{(2)nm}(t)$ are new unknown coordinates, not as for corresponding linear system, but new eigen time functions corresponding to eigen amplitude nm-modes for the transversal vibrations of the double plate homogeneous fractional order system, as solutions of the previous system (122a), which contain two separate ordinary fractional order differential equations along only one coordinate $\xi_{(1)nm}(t)$ or $\xi_{(2)nm}(t)$ but same type. From this system (122a), we can conclude that for free creep vibrations system containing two same types of ordinary fractional order differential equations as it is (11) or (40) or 74) obtained in the previous paragraphs of this paper. Then we can conclude that solutions of system (122a) equations when external plate excitations are equal to zero $f_{(1)nm}(t)-=0$ and $f_{(2)nm}(t)=0$ are normal modes of time functions with creeping properties in the vicinity of pure periodical modes of corresponding linear modes in the eigen nm-mode. Then also for the free of external excitation double plate system time modes $\xi_{(1)nm}(t)$ and $\xi_{(2)nm}(t)$ of the corresponding eigen time functions $T_{(1)nm}(t)=\xi_{(1)nm}(t)+\xi_{(2)nm}(t)$ and

$T_{(2)nm}(t)=\xi_{(1)nm}(t)-\xi_{(2)nm}(t)$ in the eigen amplitude nm -mode, .is easy to obtain by expression (99) with spittoon corresponding two eigen frequencies $\omega_{i,nm}$, $i=1,2$ of the corresponding linear system and d corresponding eigen numbers of creep system properties $\omega_{i\alpha,nm}$, $i=1,2$ in the eigen nm -mode.

5. 4* Concluding remarks for the double plate fractional order system,

The two coupled partial fractional order differential equations of transversal vibrations of a creeping connected double plates system have been derived. The analytical solutions of system coupled partial fractional order differential equations of corresponding dynamical free and forced processes are obtained by using method of Bernoulli's particular integral and Laplace's transform method.
For analysis we can compare Laplace's transform for the case of coupled plates (115) and (116) or (119) and (119a) and for uncoupled plates for creep system in the form:

$$\mathfrak{L}\{T_{(i)nm}(t)\}=\frac{pT_{0(i)nm}+\dot{T}_{0(i)nm}}{p^2\left[1+\frac{\omega_{(i)\alpha nm}^2}{p^2}\left(p^\alpha+\frac{\omega_{(i)nm}^2}{\omega_{(i)\alpha nm}^2}\right)\right]} \quad i=1,2\ ,\ n,m=1,2,3,4,....\infty \quad (123)$$

with determinant (120) or (120a) and also corresponding:

$$\Delta_{(i)nm}(p)=p^2\left[1+\frac{\omega_{(i)\alpha nm}^2}{p^2}\left(p^\alpha+\frac{\omega_{(i)nm}^2}{\omega_{(i)\alpha nm}^2}\right)\right] \quad i=1,2\ ,\ n,m=1,2,3,4,....\infty \quad (124)$$

and for ideal elastic system when $\alpha=0$, and we can conclude the following: It is shown that the two-frequency-like regime corresponds to one mode vibrations for free vibrations induced by initial conditions. Analytical solutions show us that creeping connection between plate caused appearance of like two-frequency regimes of time function corresponding to one eigen amplitude function of one mode, and also that time functions of different mn -family vibration modes $n,m=1,2,3,4,....\infty$ uncoupled.
It is shown for every shape of vibrations. It is proven, that in one of the mn -family vibration modes $n,m=1,2,3,4,....\infty$ of the both creep connected plates two possibilities for appearance of the resonance-like dynamical states are present, and also for appearance of the dynamical absorption-like, which is also similar to the appearance of the resonance and second to the dynamical absorption.

6 MULTI PLATE SYSTEM

6.1* Theoretical Problem Formulation and Governing Equations.

For the case that we have a multi plate system, let us suppose that plates are thin and that it is not the case of deplanation of cross sections in the conditions of the creep material. Also, we suppose that cross sections are always orthogonal with respect to the middle surface (plane) of the plate. If thin plates are creep bent with small deflection, i.e., when the deflection of the middle surface is small compared to the thickness h, the same assumption can be made for both plates as in the [Hedrih, 2004c, 2005].

Now, let us consider finite number M of isotropic, creeping, thin plates, width h_i, $i = 1,2,...,M$, modulus of elasticity E_i, Poisson's ratio μ_i and shear modulus G_i, plate mass distribution ρ_i. The plates are of constant thickness in the z-direction (see Fig. 8). The contours of the plates are parallel. Plates are interconnected by corresponding number $M-1$ creeping layers with the fractional order derivative constitutive relations type with constant surface stiffnesses. These creep-layers connected multiple plate systems are of composite structure type, or of sandwiched plates, or of layered plates.

The origins of the corresponding number M coordinate systems are M corresponding sets at the corresponding centres in the nondeformed plates middle surfaces as shown in Fig. 1. and with parallel corresponding axes. The plates may be subjected to either a transversal distributed external loads $q_i(x,y,t)$, $i = 1,2,...,M$ along corresponding plates external surfaces. The problem at hand is to determine solutions.

The use of Love-Kirchhoff approximation makes classical plate theory essentially a two dimensional phenomenon, in which the normal and transverse forces and bending and twisting moments on plate cross sections [see book by Rašković, 1965; 1985] can be found in term of the displacement $w_i(x,y,t)$, $i = 1,2,...,M$ of the middle surface points, which is assumed to be a function of two coordinates, x and y and time t, as in the two previous chapters.

The plates are assumed to be with same contour forms and boundary conditions.

Let us denote with $\mathfrak{D}_i = \dfrac{\mathbf{E}_i h^3}{12(1-\mu^2)}$, $\mathfrak{D}_{i\alpha} = \dfrac{\mathbf{E}_{i\alpha} h^3}{12(1-\mu^2)}$, $i = 1,2,...,M$ corresponding bending cylindrical rigidity of creep plates. For homogeneous and isotropic plates material with parameters of material creep properties are equal $\alpha_x = \alpha_y = \alpha$; also, coefficients of rigidity of momentaneous and prolongeous one are: $\mathbf{E}_{0x} = \mathbf{E}_{0y} = \mathbf{E}_0$ and $\mathbf{E}_{\alpha x} = \mathbf{E}_{\alpha y} = \mathbf{E}_\alpha$ in all directions at corresponding point. Coefficients of rigidity of momentaneous and prolongeous one for creep layer are c and c_α, and the parameter of layer material creep properties is $0 \le \alpha \le 1$.

Now, by using results of [(Stevanović) Hedrih, 2003; 2004], the relation between stress components and strain components expressed by corresponding transversal displacements $w(x,y,t)$ of the corresponding plate middle surface corresponding point $N(x,y,0)$ and coordinate z of the corresponding plate point $N(x,y,z)$, than we can write the system of M governing coupled partial fractional order differential equations of the creep connected multi plate system dynamics formulated in terms of M unknowns: the transversal displacement $w_i(x,y,t)$,

$i = 1,2,...,M$ in direction of the axis z, of the plate middle surfaces (see Figure 8) in the following form.

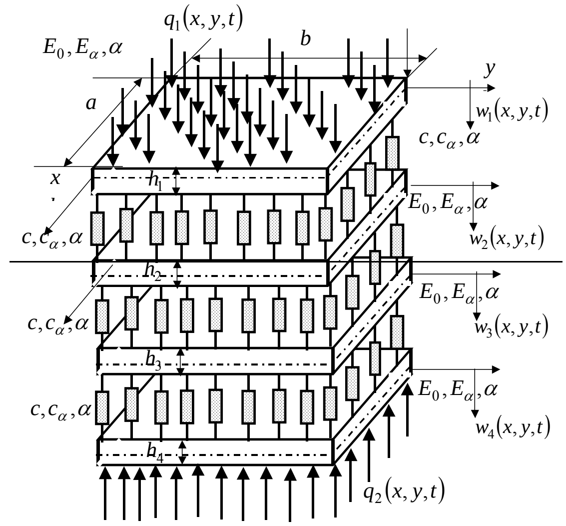


Figure 8. A creeping connected multiple plate system

$$\begin{aligned}
&\frac{\partial^2 w_1(x,y,t)}{\partial t^2} + c_{(1)}^4 \left\{\left(1+\kappa_\alpha \mathfrak{D}_t^\alpha\right)\left[\Delta\Delta w_{(1)}(x,y,t)\right]\right\} - \\
&- a_{(1)}^2 \left\{\left(1+\kappa_\alpha^c \mathfrak{D}_t^\alpha\right)\left[w_2(x,y,t) - w_1(x,y,t)\right]\right\} = \tilde{q}_1(x,y,t) \\
&\frac{\partial^2 w_i(x,y,t)}{\partial t^2} + c_{(i)}^4 \left\{\left(1+\kappa_\alpha \mathfrak{D}_t^\alpha\right)\left[\Delta\Delta w_i(x,y,t)\right]\right\} + \\
&\qquad + a_{(i)}^2 \left\{\left(1+\kappa_\alpha^c \mathfrak{D}_t^\alpha\right)\left[w_i(x,y,t) - w_{i-1}(x,y,t)\right]\right\} - \\
&\qquad - a_{(i)}^2 \left\{\left(1+\kappa_\alpha^c \mathfrak{D}_t^\alpha\right)\left[w_{i+1}(x,y,t) - w_i(x,y,t)\right]\right\} = -\tilde{q}_i(x,y,t) \\
&\frac{\partial^2 w_M(x,y,t)}{\partial t^2} + c_{(M)}^4 \left\{\left(1+\kappa_\alpha \mathfrak{D}_t^\alpha\right)\left[\Delta\Delta w_M(x,y,t)\right]\right\}_- \\
&+ a_{(M)}^2 \left\{\left(1+\kappa_\alpha^c \mathfrak{D}_t^\alpha\right)\left[w_M(x,y,t) - w_{M-1}(x,y,t)\right]\right\} = -\tilde{q}_M(x,y,t)
\end{aligned} \tag{125}$$

where $\mathfrak{D}_t^\alpha[\bullet]$ is differential operator with fractional order derivative expressed by (2) [see Enelund, 1996 and Gorenflo and Mainardi, 2000], defined by material parameter α, which satisfy the following condition: $0 < \alpha < 1$, $\tilde{q}_i(x,y,t)$ external distributed transversal loads along corresponding plate corresponding contour surfaces : $c_{(i)}^4 = \frac{\mathfrak{D}_i}{\rho_i h_i} = \frac{\mathbf{E}_0 h^3}{12\rho h(1-\mu^2)} = c_0^4$, $\kappa_\alpha = \frac{\mathbf{E}_\alpha}{\mathbf{E}_0}$, $\kappa_\alpha^c = \frac{c_\alpha}{c}$, $a_{(i)}^2 = \frac{c}{\rho h} = a_0^2$, $i = 1,2,...,M$.

6.2* Method Solution of the Governing Equations.

Solution of previous partial fractional order differential equations (125) can be looked for by using Bernoulli's method of particular integrals in the form of multiplication of two functions as in the two previous paragraphs and in some of published papers by [Hedrih, 2003; 2004], from which the first $\mathbf{W}_{(i)}(x,y)$, $i=1,2,...,M$ depends only on space.

For the corresponding boundary conditions, the space coordinate own amplitude functions we denote with $\mathbf{W}_{(i)nm}(x,y)$, $i=1,2,...,M$, $n,m=1,2,3,4,....\infty$. These eigen amplitude functions $\mathbf{W}_{(i)nm}(x,y)$, $i=1,2,...,M$, $n,m=1,2,3,4,....\infty$ satisfy boundary conditions, and also orthogonality conditions. Also, the corresponding time functions $\mathbf{T}_{(i)nm}(t)$, $i=1,2,...,M$, $n,m=1,2,3,4,....\infty$, are expressed in the form of series as it is known. k_{nm} are own characteristic number defined as a series of roots of characteristic equations obtained from corresponding plate boundary conditions, as it is known and shown in the .one of previous paragraph.

The solutions of the governing system of the corresponding coupled partial fractional order differential equations (1), we take in the eigen amplitude functions $\mathbf{W}_{(i)nm}(x,y)$, $i=1,2,...,M$, $n,m=1,2,3,4,....\infty$ expansions, from solution of the previous basic problem with decoupled equations, and with time coefficients in the form of unknown time functions $T_{(i)nm}(t)$, $n,m=1,2,3,4,....\infty$, $i=1,2,...,M$ describing their time evolution:

$$w_i(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\mathrm{W}_{(1)nm}(x,y)\mathrm{T}_{(i)nm}(t),\; i=1,2,...,M \tag{126}$$

Than after introducing the (120) into the governing system of coupled partial fractional order differential equations for free and also for forced double plates oscillations (125) and by multiplying all equations of the system with $\mathrm{W}_{(i)sr}(x,y)dxdy$ and after integrating along all surface of the plate middle surface and taking into account orthogonality conditions and corresponding equal boundary conditions of the plates, we obtain the mn-family of the systems containing coupled only two ordinary fractional order differential equations for determination of the unknown time functions $T_{(i)nm}(t)$, $i=1,2,...,M$, $n,m=1,2,3,4,....\infty$ in the following form:

$$\ddot{\mathrm{T}}_{(1)nm}(t)+\omega_{(1)nm}^2\left(1+\tilde{\kappa}_{\alpha(1)nm}\mathfrak{D}_t^{\alpha}\right)\mathrm{T}_{(1)nm}(t)-$$
$$-\left(a_{(1)}^2+a_{(1)\alpha nm}^2\mathfrak{D}_t^{\alpha}\right)\mathrm{T}_{(2)nm}(t)=f_{(1)nm}(t)$$

$$\ddot{\mathrm{T}}_{(i)nm}(t)+2\omega_{(i)nm}^{2}\left(1+\tilde{\kappa}_{\alpha(i)nm}\mathfrak{D}_{t}^{\alpha}\right)\mathrm{T}_{(i)nm}(t)- \\ -\left(a_{(i)}^{2}+a_{(i)\alpha nm}^{2}\mathfrak{D}_{t}^{\alpha}\right)\left[\mathrm{T}_{(i-1)nm}(t)+\mathrm{T}_{(i+1)nm}(t)\right]= f_{(i)nm}(t) \tag{127}$$

$$\ddot{\mathrm{T}}_{(M)nm}(t)+\omega_{(M)nm}^{2}\left(1+\tilde{\kappa}_{\alpha(M)nm}\mathfrak{D}_{t}^{\alpha}\right)\mathrm{T}_{(M)nm}(t)- \\ -\left(a_{(M)}^{2}+a_{(M)\alpha nm}^{2}\mathfrak{D}_{t}^{\alpha}\right)\mathrm{T}_{(M-1)nm}(t)=-f_{(M)nm}(t) \\ i=2,...,M-1$$

where time known function $f_{(i)nm}(t)$, $i=1,2,...,M$ are defined by following expressions:

$$f_{(i)nm}(t)=\frac{\int_0^a\int_0^b\tilde{q}_i(x,y,t)\mathrm{W}_{(i)nm}(x,y)dxdy}{\int_0^a\int_0^b\left[\mathrm{W}_{(i)nm}(x,y)\right]^2dxdy},\quad i=1,2,...,M \tag{129}$$

The system of coupled ordinary fractional order differential equations (128) on unknown time-functions $T_{(i)nm}(t)$, $i=1,2,...,M$, $n,m=1,2,3,4,....\infty$, can be solved applying Laplace transforms as in the case of the double plate system.

7 CONCLUDING REMARKS

M coupled partial fractional order differential equations of transversal vibrations of a creeping connected multi plate system have been derived. Analytical solutions of a system of M coupled partial fractional differential equations of corresponding dynamical free and forced processes are obtained by using method of Bernoulli's particular integral and Laplace transform method.

Also we can consider these ordinary fractional order differential equations (127) by new coordinates $\xi_{(s)nm}(t)$, $s=1,2,...,M$, $n,m=1,2,3,4,....\infty$ analogous as in the paragraph VIII.3* by using expressions of the dependence between generalised and main coordinates of the corresponding linear system for free vibrations when $\tilde{q}_i(x,y,t)=0$, and corresponding eigen frequencies of this linear system. Then we can transform the system (127) in the form:

$$\ddot{\xi}_{(s)nm}(t)+\left(\omega_{s,nm}^{2}+\omega_{s\alpha,nm}^{2}\mathfrak{D}_{t}^{\alpha}\right)\left[\xi_{(s)nm}(t)\right]= \\ =G_{L(s)nm}\left[f_{(1)nm}(t),f_{(s)nm}(t),.....,f_{(M)nm}(t)\right] \\ s=1,2,...,M,\quad n,m=1,2,3,4,...\infty \tag{129}$$

where $\xi_{(s)nm}(t)$, $s=1,2,...,M$, $n,m=1,2,3,4,....\infty$ are new unknown normal coordinates of the system, not as for corresponding linear system, but new eigen time functions corresponding to eigen amplitude nm -modes for transversal vibrations of the double plate homogeneous fractional order system, as solutions of the previous system (127), which contain M separate ordinary fractional order differential equations along only one coordinate from the set $\xi_{(s)nm}(t)$, $s=1,2,...,M$, $n,m=1,2,3,4,....\infty$ but of the same type. $G_{L(s)nm}\left[f_{(1)nm}(t), f_{(s)nm}(t),....., f_{(M)nm}(t)\right]$ are linear combination of functions $f_{(i)nm}(t)$, $i=1,2,...,M$, depending on the corresponding functional form between coordinates – time functions $\xi_{(s)nm}(t)$, $s=1,2,...,M$, $n,m=1,2,3,4,....\infty$ and $T_{(i)nm}(t)$, $i=1,2,...,M$, $n,m=1,2,3,4,....\infty$ in the starting of the coordinate substitution and transformation of the system of ordinary fractional order equations from (127) to (129).
From this system (129), we can conclude that for free creep vibrations system contains M number of the same type of ordinary fractional order differential equations as it is (11) or (40) or (74) obtained in the previous paragraphs of this paper. Then we can conclude that solutions of the system (129) of equations when external plate excitations are equal to zero $\tilde{q}_i(x,y,t)=0$ are M normal modes of the time functions $T_{(i)nm}(t)$, $i=1,2,...,M$, $n,m=1,2,3,4,....\infty$ with creeping properties in the vicinity of the pure periodical modes of the corresponding linear modes in the eigen nm -mode.
By using trigonometric method and solution of the system of algebra equations in the form $A_k = C\sin k\varphi$, we can obtain that $\varphi_s=\frac{s\pi}{M}$, $s=1,2,3,4,....,(M-1)$, and that the determinant $\Delta_{nm}(p)$ of the nm-family of the system equations obtained by Laplace transform of the system equations (128) can be obtained as one of main results of this investigation in the form:

$$\frac{1}{\Delta_{nm}^{(M)}(p)}=\prod_{s=1}^{s=M-1}\frac{1}{\left\{\left\langle p^2+2\omega_{nm}^2\left[1+\frac{\omega_{\alpha\,nm}^2}{\omega_{nm}^2}p^\alpha\right]\right\rangle-2a^2\left[1+\frac{a_\alpha^2}{\omega^2}p^\alpha\right]\cos\frac{s\pi}{M}\right\}}$$

This form is suitable for obtaining separate simple members of expression of solutions with simple way of the inverse Laplace transform application.
Analytical solutions show us that creeping connection between plates in M -multi plate system caused appearance of like M -frequency regimes of time function corresponding to one eigen amplitude function of one nm -mode, and also that time functions of different mn -family vibration modes $n,m=1,2,3,4,....\infty$ are uncoupled for considered multi plate system.

It is shown for every shape of vibrations. It is proved that in one of the mn-family vibration modes $n,m=1,2,3,4,....\infty$ of the all M-creep connected plates are present M possibilities for appearance of the resonance-like dynamical states, and also for appearance of the dynamical absorption-like.
Chain dynamics of the homogeneous system – sandwich multi beam, multi plate systems as well as multi pendulum system are investigated by using mathematical analogy and phenomenological mapping.

Acknowledgment. Part of this research was supported by the Ministry of Sciences of the Republic of Serbia through Mathematical Institute SANU Belgrade Grants No. ON144002 Theoretical and Applied Mechanics of the Rigid and Solid Body. Mechanics of Materials, and also through Faculty of Mechanical Engineering University of Niš.

REFERENCES

[1] Ariaratnam S. T., (1996), Stochastic Stability of Viscoelastic Systems under bounded Noise Excitation, IUTAM Symp. on Advances in Nonlinear Stochastic Mechanics, Kluwer Acad. Publ. Dordrecht, 1996, pp. 11-18.
[2] Betten, J. Mathematical modeling of materials behavior under creep conditions, ASME Applied Mechanics Reviews, vol. 54, no. 2 March 2001., pp. 107-132.
[3] Bishop R.E.D., (1952), Longitudinal Waves in Beams, Aero. Q., 3, 280-293.
[4] Bačlić, B. S., Atanacković, T. M., Stability and creep of a fractional derivative order viscoelastic Rod, Bulletin T, CXXI de L'Academie Serbe des Sciences st de Arts - 2000, Class des Sciences mathematques et naturelles Sciences mathematiques, No. 25, 115-131.
[5] Dli Gen-guo, Zhu Zheng-you and Cheng Chanh-jun, (2001), Dynamical Stability of viscoelastic Column with Fractional Derivative Constitutive Relation, Applied Mathematics and Mechanics, English Edition, vol. 22. No. 3, Mar. 2001, Shanghai, China, pp. 294-303.
[6] Enelund, M., Fractional Calculus and Linear Viscoelasticity in Structural Dynamics, Division of Solid Mechanics, Chalmers tekniska Hogskola, Goteborg, Sweden, 1996., p.1-27, A1-33+B1-20+C1-19+D1-28+E1-26.
[7] Gorenflo, R., Mainardi, F., (2000) Fractional Calculus, Integral and Differential Equations of Fractional Order, CISM Lecture Notes, Udine, Italy, Preprint 54 pages, pp. 223-276.
[8] Goroško O. A. i Hedrih (Stevanović) K., Analitička dinamika (mehanika) diskretnih naslednih sistema, (Analytical Dynamics (Mechanics) of Discrete Hereditary Systems), University of Niš, 2001, Monograph, p. 426, YU ISBN 86-7181-054-2.
[9] Goroško O. A. i Hedrih (Stevanović) K.,, Construction of the Lagrange's Mechanics of the hereditary systems,, Facta Universitatis Series Mechanics, Automatic Control and Robotics, Vol .6, No. 1 2007, pp. 1-23.

[10] O. Goroshko, K. Hedrih. Dynamics of the hereditary discrete systems, Proceedings Nonlinear Dynamics , Polytechnic Kharkov, 2008. ,pp. 77-82
[11] Goroško O. A. and Hedrih (Stevanović) K., (2008), Advances in development of the analytical dynamics of the hereditary discrete systems, Journal of Physics: Conference Series, 96(2008) 012143, http://www.iop.org/EJ/main/-list=current/
[12] Goroshko O.A. and Hedrih (Stevanović), K., (2007), Construction of the Lagrange mechanics of the hereditary systems, Minisymposium Oppening Lecture, Proceedings of the International Summer School APM –Advanced Problem in Mechanics, Saint Petersburg 2007., pp. 133-156. (Minisymposia IDS - OpeningLlecture)
[13] Goroshko, O.A., Puchko, N.P., Lagrangian equations for the multibodies hereditary systems, Facta Universitatis, Series Mechanics, Automatic Control and Robotics, Vol. 2, No 7, 1997, pp. 209-222.
[14] Hedrih (Stevanović) K., Thermorheological Hereditary Pendulum, (Ref. No. TVP-11) Thermal Stresses 99, Edited by J.J. Skrzypek and R. B. Hetnarski, Cracow 1999, pp.199-202.
[15] Hedrih (Stevanović) K., Transverzalne oscilacije grede od naslednog materijala - Prvi deo. Parcijalna integro-diferencijalna jednačina, Originalni naučni rad, Tehnika, Mašinstvo 49 (2000) 1, str. M1-8M..
[16] Hedrih (Stevanović) K., Transverzalne oscilacije grede od naslednog materijala - drugi deo. Primena ideje S.T. Ariaratnama za ispitivanje stabilnosti formi grede pod dejstvom slučajnih sila, Originalni naučni rad, Tehnika, Mašinstvo 49 (2000) 2, str. M1-6M.
[17] Hedrih (Stevanović) K., (2000), Dynamics of Discrete Hereditary Chain Systems, (in Macedonian), 7-th Symposium on Theoretical and Applied Mechanics, Struga, Macedonia, pp. 245-254.
[18] Katica (Stevanović) Hedrih, Transversal Creep Vibrations of a Beam with Fractional Derivative Constitutive Relation Order, First part: Partial Fractional-differential equation. Part second: The S.T. Ariaratnam idea Applied to the Stochastic Stability of the Beam Dynamic Shape, under Axial Bounded Noise Excitation, Proceedings of Forth International Conference on Nonlinear Mechanics (ICNM-IV), edited by Wei Zang Chien and All, August 14-17, 2002, Shanghai, P.R. China, pp. 584-595. . ISBN 7-81058-077-9/0-007.
[19] Hedrih (Stevanović) K., The transversal creep vibrations of a fractional derivative order constitutive relation on nonhomogeneous beam, XVIII Yugoslav Conference with International Participation-Noise and Vibrations, Niš 2002, 17-18 October, 2002., Plenary Lecture, Abstracts p.55, Full paper in CD-Proceedings pp.1-13.
[20] Katica (Stevanović) Hedrih, (2008), Energy transfer in double plate system dynamics, Acta Mechanica Sinica, Volume 24, Number 3 / June, 2008, pp. 331-344, DOI 10.1007/s10409-007-0124-z, Springer Berlin / Heidelberg, ISSN (567-7718 (Print) 1614-3116 (Online).
[21] Hedrih (Stevanović) K., (2006), Modes of the Homogeneous Chain Dynamics, Signal Processing, Elsevier, 86(2006), 2678-2702.. ISSN: 0165-1684 www.sciencedirect.com/science/journal/01651684

[22] Hedrih (Stevanović) K., (2005), Integrity of Dynamical Systems, Journal Nonlinear Analysis, 63 (2005) 854 – 871.

[23] Hedrih (Stevanović), K., (2007), Dynamics of coupled systems, Journal Nonlinear Analysis: Hybrid Systems, Elsevier, Volume 2, Issue 2, June 2008, Pages 310-334.

[24] Hedrih (Stevanović), K., (2007), Energy analysis in the nonlinear hybrid system containing linear and nonlinear subsystem coupled by hereditary element, Nonlinear Dynamics, Springer, 30.01.2007, vol. 51, no. 1, pp. 127-140. DOI 10.1007/s11071-007-9197-2.

[25] Hedrih (Stevanović), K., (2006b), Transversal vibrations of the axially moving sandwich belts, Archive of Applied Mechanics, Springer, 03.02.2007, vol. 77, no. 7, pp. 523-539. DOI 10.1007/s00419-007-0187-0 http://springerlink.com/content/?k=Hedrih

[26] Hedrih (Stevanović) K., (1997), Energy Analysis of Oscillatory Processes and the Modes in Nonlinear Systems, Solid Mechanics, Serbian-Greek Symposium 1997, Scientific Meetings of the Serbian Academy of Sciences and Arts, Department of Technical Sciences, SANU Belgrade, vol. LXXVII., Book. 3, pp. 137-146.

[27] Hedrih (Stevanović) K., (1997), Interpretation of the transfer of energy from high-frequency to low-frequency modes by averaging asymptotic method Krilov-Bogolybov-Mitropolsky, AiM 96, Asymptotics in Mechanics, Proceedings of the Second International Conference, Edited by Ali H. Nayfeh and Kiril V. Rozdestvensky, p. 302, Saint-Petersburg State Marine Technological University, Saint Petersburg, 1997, pp. 97-104. ISBN 5-88303-069-6, Izdatelskiy centr SPbGMTU, 1997.

[28] Hedrih (Stevanović) K., (2006), Transversal Vibrations of Double-Plate Systems, Acta Mechanica Sinica, Springer, (2006) 22, pp. 487-501 (hard cower and on line).

[29] Hedrih (Stevanović), K., (2007), Double plate system with discontinuity in the elastic bonding layer, Acta Mechanica Sinica VOL. 23, NO. 2, PP. 221-229. (hard cower and on line).

[30] Hedrih (Stevanović), K., (2006), The frequency equation theorem of small oscillations of a hybrid system containing coupled discrete and continuous subsystems, Facta Universitatis Series: Mechanics, Automatic Control and Robotics, Vol.5, No 1, 2006 pp. 25 - 41 UDC 534.1:534.012:534.013: http://facta.junis.ni.ac.yu/facta/

[31] Hedrih (Stevanović), K., (2007), Energy analysis of the double plate system, Acta Mechanica Sinica, Springer, DOI 10.1007/s10409-007-0124-z

[32] Hedrih (Stevanović) K., (2003), Frequency equations of small oscillations mixed systems of the coupled discrete and continuous subsystems, Mehanika tverdogo tela (Rigid Body Mechanics), Donetsk, UDC 531.1:534.012:534.013:, ISSN 0321-1975, vip. 33, pp. 174-189. Ukraine.

[33] Hedrih (Stevanović), K., (2005), Eigen Amplitude Vectors and Functions Extended Orthogonality of Small Oscillations Mixed Systems of the Coupled Discrete and Continuous Subsystems, Facta Universitatis, Series Mechanics, Automatic Control and Robotics, Vol. 4 No. 17, 2005. pp. 225-243. YU ISSN

0534-2009. UDC 534.1:534.012:534.013.http://facta.junis.ni.ac.yu/facta/macar/macar200501/macar200501-04.html
[34] Hedrih (Stevanović) K., (2005), Partial Fractional Differential Equations of Creeping and Vibrations of Plate and their Solutions (First Part), Journal of the Mechanical Behavior of Materials, Freund Publishing House Ltd. (accepted for publishing).2005, pp. 305-314. ISSN 0334-8938

http://www.freundpublishing.com/JOURNALS/materials_science_and_engineering.htm
[35] Hedrih (Stevanović), K.,, (2006), The frequency equation theorems of small oscillations of a hybrid system containing coupled discrete and continuous subsystems, Facta Universitatis, Series Mechanics, Automatic Control and Robotics, Vol.5, No 1, 2006 pp. 25 – 41. UDC 534.1:534.012:534.013 http://facta.junis.ni.ac.yu/facta/
[36] Hedrih (Stevanović), K., (1995), Energijska analiza kinetike konstrukcija za razlicite modele materijala (Energy analysis of the sytucture kinetics for different models of the materials), Kratki prikazi radova, "Mehanika, materijali i konstrukcije", SANU, Beograd, 1995, pp. 106-107. (in Serbian)
[37] Hedrih (Stevanović) K., (2005b), Multifrequency forced vibrations of thin elastic Shell, CD Proceedings, Fifth EUROMECH Nonlinear Dynamics Conference, Eindhoven University of Technology, Editors: D.H. van Campen, M.D. Lazurko, W.P.J.M. van den Over, ID of contribution 21-439, pp2417-2426. ISBN 90 386 2667 3, www.enoc2005.tue.nl
[38] Hedrih (Stevanović) K., (2005c), Partial Fractional Order Differential Equations of Transversal Vibrations of Creep-connected Double Plate Systems, in Monograph - Fractional Differentiation and its Applications, Edited by Alain Le Mahaute, J. A. Tenreiro Machado, Jean Claude Trigeassou and Jocelyn Sabatier, U-Book, pp. 289-302.
[39] Hedrih (Stevanović) K.,The Dissipation Function of a Nonconservative System of Mass Particles, Tensor, N.S.,Vol.63, No.2(2002), pp.176-186. ISSN 0040-3504
[40] Hedrih (Stevanović), K., (2007), Hybrid Systems and Hybrid Dynamics: Theory and Applications, Invited Plernary Lecture, 8th HSTAM International Congress on Mechanics, Patras, 12 – 14 July, 2007, Greece, Proceedings, Edited by N. Bazwos, D.L. Karabalis, D. Polyzos, , D.E. Beskos and J. T. Katsikadelis, Volume I, pp. 77-86.
[41] Hedrih (Stevanović), K., Discrete Continuum Method, Symposium, Recent Advances in Analytical Dynamics Control, Stability and Differential Geometry, Proceedings Mathematical institute SANU Edited by Vladan Djordjević, p. 151, 2002, pp.30-57. .ISBN 86-80593-32-X.
[42] Hedrih (Stevanović) K., (2004), Discrete Continuum Method, COMPUTATIONAL MECHANICS, WCCM VI in conjunction with APCOM'04, Beijing, China, © 2004 Tsinghua University Press & Springer-Verlag, pp. 1-11, CD.

[43] Hedrih (Stevanović) K., (2002), Transversal Creep Vibrations of a Beam with Fractional Derivative Constitutive Relation Order, First part: Partial Fractional-differential equation . Part second: Stochastic Stability of the Beam Dynamic Shape, under Axial Bounded Noise Excitation, Proceedings of Forth International Conference on Nonlinear Mechanics (ICNM-IV), edited by Wei Zang Chien and All, August 14-17, 2002, Shanghai, P.R. China, pp. 584-595.

[44] Katica (Stevanović) Hedrih, Differential Equations of Two Mass Particles, Constrained with a Piezo-thermo-rheological Hereditary Element, Dynamics, Proceedings of full papers, Edited by Dragutin Veličković, 5 th International Conference on Applied Electromagnetics, PES 2001., pp. 77-80.

[45] Hedrih (Stevanović) K., Discrete Continuum's Models and Thermo-rheological Elements - Basic Idea and Tensor Equations, Homogeneous Linear Chain and Plane/Space Material Nets,Proceedings of full papers, Edited by Dragutin Veličković, 6 th International Conference on Applied Electromagnetics, PES 2003, Stability Pact for South Eastern Europe, DAAD, pp. 127-130 and pp. 131-134.

[46] Hedrih (Stevanović), K., Hybrid System Dynamics, Proceedings, First International Congress of Serbian Society of Mechanics, Editors: D. Šumarac and D. Kuzmanović, Srpsko društvo za mehaniku, 10-13, April, 2007, pp. 79-84. http://www.ssm.org.yu/congress2007/authors.html

[47] Hedrih (Stevanović), K., (2007), Multifrequency and infinite frequency regimes and Lyapunov exponents as the measures of the hybrid system dynamic integrity, Proceedings of the International Summer School APM –Advanced Problem in Mechanics, Saint Petersburg 2007., pp. 157-169. (Minisymposia IDS – Closing Llecture)

[48] Hedrih (Stevanović), K., (2007), Stochastic dynamics of hybrid systems with thermo-rheological hereditary elements, Edited by Awrejcewicz P. Olejnik abd Mrozowski, Proceedings of IX INTERNATIONAL CONFERENCE ON DYNAMICAL SYSTEMS - THEORY AND APPLICATIONS, December 17-20, 2007. Łódź, POLAND, Technical University of Łódź. Department of Automatics and Biomechanics., Abstracts p. 62, Full Paper, pp.193-202.

[49] Hedrih (Stevanović) K., Izabrana poglavlja Teorije elastičnosti, Mašinski fakultet u Nišu, (Same Chapters in Theory of Elasticity), prvo izdanje 1976, drugo izdanje 1988, str. 424.

[50] Hedrih (Stevanović), K., (1999), Rheonomic Coordinate Method Applied to Nonlinear Vibration Systems with Hereditary Elements, www-EUROMECH, Copenhagen 1999, http://www.imm.dtu.dk/documents/user/mps/ENOC/Proceedings/ Technical University of Denmark, 4A, pp.1-24.

[51] Hedrih (Stevanović) K., (2006), Transversal Vibration of a Parametrically Excited Beam: Influence of Rotatory Inertia and Transverse Shear on Stochastic Stability of Deformable Forms and Processes, ©Freund Publishing House Ltd. International Journal of Nonlinear Sciences and Numerical Simulation, 7(1), 117-124, 2006. ISSN 1565-1339.

[52] Hedtrih (Stevanović) K., (2008), Dynamics of coupled systems, Nonlinear Analysis: Hybrid Systems, Volume 2, Issue 2, June 2008, Pages 310-334.
[53] Hedtrih (Stevanović) K., (2008a), Thermo-rheological hereditary elements and discrete continuum model dynamics, "Mashinoznavstvo", Ukraine, # 2 (128) za 2008, pp. 18 - 27. (in English).
[54] Hedrih (Stevanović), K., (2008c), Analogy between dynamics of thermo-rheological and piezo-rheological pendulums, Journal of Physics: Conference Series, 96 (2008) 012219 doi:10.1088/1742-6596/96/1/012219, IOP Publishing, http://www.iop.org/EJ/main/-list=current/
[55] Hedrih (Stevanović) K., (2002b), The Dissipation Function of a Nonconservative System of Mass Particles, Tensor, N.S.,Vol.63, No.2(2002), pp.176-186. ISSN 0040-3504.
[56] Hedrih (Stevanović) K., (2003b), The Longitudinal and Transversal Creep Vibrations of a Fractional Order Constitutive Relation Beams, Scientific Bulletin of the Politehnica, University of Timisoara, Transaction on Mechanics, Tom 48(62), Fasc. 1, 2003. pp. 5-12, 13-22, Editura Politehnica, Timisoara. ISSN 1224-6077. http://www.utt.ro/english/pbseng.shtml
[57] Hedrih (Stevanović) K., (2004), Partial fractional order differential equations of transversal vibrations of creep connected double plates systems, Workshop Preprints/Proceedings No 2004-1 IFAC workshop on Fractional Differentiation and its Applications FDA 04, ENSEIRB, Bordeaux France, July 19-21, 2004., pp. 299-304.
[58] Hedrih (Stevanović) Katica, (2004a), Creep Vibrations of a Fractional Derivative Order Constititive Relation Deformable Bodies, Proceedings Eighth American Congress of Applied Mechanics PACAM VIII. La Habana, Cuba, 5-9 January 2004, Series Applied Mechanics in Americas, Vol. 10. pp. 548-551, ISBN 959-7056-20-8.
[59] Hedrih (Stevanović), K., (2006c), The transversal creeping vibrations of a fractional derivative order constitutive relation of nonhomogeneous beam, Mathematical Problems in Engineering, Special issue : Nonlinear Dynamics and their Applications in engineering sciences, Geust Editor: Jose Manoel Barhesar, Volume 2006 (2006), Article ID 46236, 18 pages, www.hindawi.com, doi:10.1155/MPE/2006/46236, Volume 2006, No. 5, pp. 61-78.
[60] Hedrih (Stevanović) K.,, (2008), Energy transfer in double plate system dynamics, Acta Mechanica Sinica, Volume 24, Number 3 / June, 2008, pp. 331-344, DOI 10.1007/s10409-007-0124-z, Springer Berlin / Heidelberg, ISSN (567-7718 (Print) 1614-3116 (Online),
[61] Hedrih (Stevanović) K.,, (2008), Energy interaction between linear and nonlinear oscillators (Energy transient through the subsystems in the hybrid system), ISSN 1027-3190. Ukr. mat. Ωurn., 2008, t. 60, # 6, pp. 796-814.
[62] Hedrih (Stevanović) K.,, (2208), Dynamically determined and undetermined hereditary discrete systems (External and internal degrees of freedom in the hybrid hereditary dynamics), Invited Lecture-Key note Lecture,The Euromech Colloquium 498 Nonlinear Dynamics of Composite and Smart Structure

(NDCS) -Nonlinear Dynamics and Chaos of Composite and Smart Structures (NDCS), May 21-23, 2008, Kazimierz Dolny, POLAND. Lublin 2008, pp. 29-39.

[63] Hedrih (Stevanović) K., Mathematical analogy between hybrid system dynamics, 45 minutes Invited Plenary Lecture, Book of Extended Abstracts, Edited by Alexander M. Kovalev (IAMM NASU, kovalev@iamm.ac.donetsk.ua), 10th International Conference "Stability, Control and Rigid Bodies Dynamics" Donetsk (Ukraine), June 2008, pp. 125-127.

[64] Katica (Stevanović) Hedrih, Leonhard Euler (1707-1783) and Rigid Body Dynamics, Scientific Technical Review, Vol. LVII, No. 3-4, Belgrade, 2007 . pp. 3-12. YU ISSN 1820-0206

[65] Hedrih (Stevanović), K., Filipovski, A., Longitudinal Vibrations of Rheological Rod with Variable Cross Section, Int. Journal Comunications in Nonlinear Sciences and Numerical Simulations, Shanghai Inst. of Applied Mathem. and Mechanics, China, 1999, vol.4, No. 3, pp. 89-95. ISSN 10007 5704 - Center for Nonlinear Science peking University, China.

[66] Hedrih (Stevanović) K. and Filipovski A., Longitudinal Creep Vibrations of a Fractional Derivative Order Rheological Rod with Variable Cross Section, Facta Universitatis, Series Mechanics, Automatics Control and Robotics, Vo. 3, No. 12, 2002, pp. 327-350. (in English).YU ISSN 0354-2009. (University Journal Facta Universitatis – Series Mechanics, Automatic Control and Robotics, University of Niš, Mathematical Reviews Annual Index 2000, Author Index M-Z, Key Index, Abbreviations of Names of Serials, . American Mathematical Society, Providence, Rhode Island, USA.)

[67] Hedrih (Stevanović K. and Simonović J., Transversal Vibrations of a Double Circular Plate System with Visco-elastic Layer Excited by a Random Temperature Field, International Journal of Nonlinear Sciences and Numerical Simulation, 2008, Vol. 9, No.1, pp.47-50. http://www.ijnsns.com/2008/TOC9.1.doc

[68] Hedrih (Stevanović) K., and Simonović J., Dynamical Absorption and Resonances in the Sandwich Double Plate System Vibration with Elastic Layer, Scientific Technical Review,Vol.LVII,No.2,2007,pp. 1-10.

[69] Hedrih (Stevanivic) K. and Simonovic J., Transversal Vibrations of a non-conservative double circular plate system, FACTA UNIVERSITATIS Series: Mechanics, Automatic Control and Roboticsos, VOL. 6, No 1, 2007, pp. 45 - 64. http://facta.junis.ni.ac.yu/macar/macar200701/macar200701toc.html

[70] Katica (StevanovIć) HEDRIH and Julijana Simonović, (2208), Non-linear dynamics of the sandwich double circular plate system, The Euromech Colloquium 498 Nonlinear Dynamics of Composite and Smart Structure (NDCS) -Nonlinear Dynamics and Chaos of Composite and Smart Structures (NDCS), May 21-23, 2008, Kazimierz Dolny, POLAND. Lublin 2008, pp. 170-175.

[71] Janković, V. S., Potić, V. P. and Hedrih (Stevanović), K., Parcijalne diferencijalne jednačine i integralne jednačine sa primenama u inženjerstvu (Partial differential equations and integro-differential equations with examples in engineering), Univerzitet u Nišu, 1999, str. 347. (in Serbian).

[72] Mathieu, E., Cours de Physique Mathématique, Paris, 1873.
[73] Mitropolyskiy, Yu. A. (1955), Nestashionarnie proshesi v nelinyeynih sistemah, AN USSR, Kiev. (in Russian)
[74] Mitropolyskiy Yu.A., Nguyen Van Dao, (2003), Lectures on Asymptotic Methods of Nonlinear Dynamics, Vietnam National University Publishing House, Hanoi, p. 494.
[75] Mitropolyskiy, Yu.A and Mosseenkov, B.I., (1968), Lekciyi po primenyeniyu metodov k recheniyu uravnyeniy v chastnih proizvodnih, Int. Math. AN USSR, Kiev. (in Russian)
[76] Mitropolyskiy, Yu.A, , (1964), Problemi asimptoticheskoy teorii nestashionarnih kolebaniy, Nauka Moskva, pp. 431. (in Russian)
[77] Mitropolyskiy, Yu. A., (1971), Metod usrednyenia v nelinyeynoy mehanike, Naukova Dumka, Kiev, pp.340.
[78] Mitropolyskiy, Yu. A., (1995), Nelinyeynaya mehanika-Asimptoticcheskie metodi, Institut matematiki NAN Ukraini, Kiev, pp. 397.
[79] Rašković, D., (1965), Teorija oscilacija, Naučna knjiga, 1965, 503.
[80] Rašković, D., (1985), Teorija elastičnosti, Naučna knjiga, 1985, 414.
[81 Rabotnov, Yu.N., (1977), Elements of Hereditary Mechanics of Solids, Moscow, Nauka (in Russian).
[82] Rzhanitsin, A.R., (1949), Some Questions of the Mechanics of Deforming in time Systems., Moscow, GTTI (in Russian). 1949, p. 248.
[83] Savin G. N., Ruschisky Yu. Ya., (1976), Elements of Hereditary Media Mechanics, Kyiv, Vyscha shkola (in Ukrainian), p. 250.
[84] Slonimsky G.L., (1961), On the Law of Deforming of High Polymer Bodies, Rep. Acad. Sci. USSR, Vol. 140, No.2 (in Russian).
[85] Stevanović (Hedrih) Katica, Application of the Asymptotic Method for study of the Elastic Bodies Nonlinear Oscillations – Energy Analysis of the Elastic Bodies Oscillatory Motions, Doctoral Dissertation, Mašinski fakultet, Niš, 1975., str. 331. (in Serbian).
[86] Torvik P. J. Torvik, Bagley, R. L., On the Appearance of the Fractional Derivatives in the Behavior of Real Materials, Journal of Applied Mechanics (Trasaction ASME) 51 (1984), pp. 294-298.
[87] Strutt, M.J. O.: Lamésche, Mathieusche und verwandle Funktionen in Physik und Technik, Berlin,1932.

Sent: Tuesday, May 25, 2008 11:56 AM