Parametric instabilities of nonlinearly viscoelastic cylindrical shells and rings subject to periodic hydrostatic excitations

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Abstract

Refined theories for accurate predictions of the onset of the parametric instabilities and the ensuing post-critical solutions are employed to investigate on how these instabilities are influenced by commonly neglected effects in shells and rods such as inertia terms, shear deformations, and material nonlinearities (Antman, 2005; Antman and Calderer, 1987; Antman, 2001).

Parametrically excited systems are pervasive in mechanics (e.g., dynamic buckling of columns, rings and shells, water waves in vertically forced containers, stability of general motions). In particular, parametrically forced rings and cylindrical shells are of significant interest in engineering applications such as aircraft fuselages or turbo machineries where the forcing is represented by the gradient between the inner and outer pressures. Although the theory of parametrically excited linear discrete systems governed by linear ordinary-differential equations is well established (Yukubovich and Starzhinskii, 1975; Nayfeh and Mook, 1979), a comprehensive theory of parametrically excited nonlinear systems is far from being achieved. Moreover, only a few works have treated parametrically excited structural systems taking into account inertia, geometric and material nonlinearities.

Among others, Bolotin (1964) studied the Mathieu equation with cubic nonlinearities while, more recently, Rand and co-workers (2004) have investigated nonlinear Mathieu equations with either quadratic damping or cubic springs. In the quadratically-damped Mathieu equation, they showed the existence of a secondary bifurcation in which a pair of limit cycles come together and disappear (a saddle-node of limit cycles). In parameter space, this secondary bifurcation appears as a curve which emanates from one of the transition curves of the linear Mathieu equation. Further, Rand (1996) studied a two-term truncation of a parametrically excited PDE, and showed, using averaging, that the normal form of the system exhibits a rich diversity of dynamical behaviors.

When dealing with parametric excitations in spatially continuous structural systems, to overcome discretization errors, that can be also qualitative in the worst scenario, there is a need to rigorously treat, e.g. via asymptotics, the governing parametrically forced PDE's instead of dealing with their reduced-order counterparts (Lacarbonara, 1999). In the present work, an asymptotic

multiple-scale treatment of parametrically forced nonlinear PDE's, representative of cylindrical shells and rods suffering bending, extension and shear deformations, is presented. Considering nonlinearly visco-elastic cylindrical shells and rings under hydrostatic pressure, general results about the leading classes of motions are discussed; namely, breathing motions, shearless motions and general motions of shells and rings undergoing extension, bending and shear deformations.

It is shown that when the shell or ring are subject to uniform pulsating pressure with frequency being nearly twice the frequency of the breathing mode, the ring or shell suffer a principal parametric resonance resulting into high-amplitude radial motions. Moreover, these motions are of the softening or hardening type depending on the constitutive law. Hence, the threshold constitutive laws delimiting softening from hardening breathing motions are obtained in closed form.

Subsequently, enforcing the inextensibilty and unshearability constraints, purely bending motions are investigated. The normal forms governing the slow-flow dynamics of the directly excited flexural mode and its companion mode are delivered by the asymptotic treatment in the case of an individual-mode instability. Again, in the space of the parameters regulating the nonlinearly elastic parts of the constitutive laws, we determined the regions where the ring is softening or hardening. In particular, when the constitutive function is linearly elastic, the flexural motions are all softening in agreement with previous analytical, numerical and experimental investigations (Evensen, 1966; Ganapathi et al., 2003).

The instabilities arising from multi-mode parametric resonances are also discussed. It is worth mentioning that in previous works (Chin et al., 1997), parametrically excited two-to-one interactions were investigated in buckled beams considering low-order geometric nonlinearities only. In this work, the prominent role of the nonlinear visco-elastic constitutive functions as well as that of the internal kinematic constraints on the parametric instabilities are highlighted within the context of a geometrically exact mechanical formulation.

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