CONSTRUCTION OF THE LAGRANGE MECHANICS OF THE HEREDITARY SYSTEMS

Oleg Alexandrovich Goroshko
Ukrainian Higher Education Academy of Sciences
Kiev National University named Taras Shevchenko, KIEV (Kiïv) 2166
Vul. Lísoviy prospekt No 22. kv. 32. (Lesnoy prospekt No 22. kv. 32.)
Tel: (044) 5181508 e-mail: p_natalia@univ.kiev.ua

Katica (Stevanović) Hedrih
Address: Katica (Stevanovic) HEDRIH, Ph.D. Professor, Dip. Mech. Eng.
Faculty od Mechanical Engineering University of Niš, Mathematical Institute SANU,
Yu-18 000 - Niš,Ul.Vojvode Tankosica 3/22, Serbia
Telefax: 381 18 41 663 Mobile 063 8 75 75 99
e-mail: katica@masfak.ni.ac.yu - e-mail(houm): khedrih@eunet.yu

In current literature term "*hereditary*" and "*rheological*" systems are equivalent. In opinion of Работнов Ю.Н. [1], the name "*hereditary*" system or continuum proposed by В.Вольтерра, is more precise as well as suitable. By use this name, the property of rheological systems "*to remember*" history of loading is fully described. By series of the fundamental papers and monographs, Mechanics of hereditary continuum is presented. Also, in numerous references, many examples with applications in engineering, biological and other area [2] are published. Mechanics of discrete hereditary systems up to a few years before was presented only by separate single papers [3] and containing only solutions of the partial problems.

Research results in area of mechanics of the hereditary discrete systems, obtained by authors of this paper, are generalized and presented in the monograph [4] which contains first presentation of the analytical dynamics of the hereditary discrete systems. We can conclude that this monograph contains complete foundation of the analytical dynamics theory of discrete hereditary systems and by using these results, numerous examples are obtained and solved (see Refs. [5-12]).

Hereditary system is every system which contains mutual hereditary interaction between material particles in the form of one or more constraints with hereditary properties.

There are three mathematical forms for description of the constitutive relations of the hereditary properties of the hereditary interaction [2], in the building of the hereditary system's mechanics. These forms are:

1. Differential equation, expressed in the form of dependence reaction force P of the rheological coordinate x, usually presented as deformation or relative displacement of the hereditary constraint:

$$\sum_{r=1}^{n} a_k \frac{d^k P}{dt_k} + P(t) = b_0 x + \sum_{k=1}^{n} b_k \frac{d^k x}{dt^k} \tag{1}$$

2. Integral equation, expressed in the form of dependence reaction force P of the rheological coordinate x, usually presented as deformation or relative displacement of the hereditary constraint:

$$P(t) = c\left(x(t) - \int_0^t R(t-\tau)x(\tau)d\tau \right) \tag{2}$$

By this integral equation, the relaxation of the reaction force P depending of the rheological coordinate x, is presented and expressed..

3. Integral equation, expressed in the form of dependence rheological coordinate x, usually presented deformation or relative displacement of the hereditary constraint and reaction force P:

$$x(t) = \frac{1}{c}\left[P(t) + \int_0^t K(t-\tau)P(\tau)d\tau \right], \tag{3}$$

By this integral equation, the retardation of the rheological coordinate x of the reaction force P is presented and expressed.

In the previous integral equations $R(t-\tau)$ and $K(t-\tau)$ - are relaxational and rheological kernel. .

Previous integral equations (2) and (3),can be expressed in following form:

$$P(t) = c\left(x(t) - \int_0^t R(\tau) x(t-\tau) d\tau \right), \tag{4}$$

$$x(t) = \frac{1}{c}\left(P(t) + \int_0^\infty K(\tau) P(t-\tau) d\tau \right), \tag{5}$$

where by integral operators, the histories of the previously interactions of the hereditary constraints are expressed.

In the basis of the construction of the Lagrange's mechanics of the hereditary discrete systems, the classical mechanics principle are used. These principles are: Principle of the work of the forces along corresponding possible system displacements, as well as Principle of dynamical equilibrium.

Using the Principle of the work of the system forces along corresponding possible system displacements we can write the following equation:

$$\sum_{i=1}^{3N} \left(m_i \ddot{x}_i - X_i + \sum_{k=1}^{K} P_k e_{ik} \right) \delta x_i = 0, \tag{6}$$

where force of rheological reaction P_k of the rheological interactions between material particles are present because constraint is no ideal.

Lagrange's equations second kind for hereditary systems are in the form:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} = \frac{\partial \Pi}{\partial q_j} + Q_j + \sum_{k=1}^{K} b_{kj} P_k, \; j = 1,2,\ldots n \tag{7}$$

From previous equations, It is necessary to eliminate reaction forces P_k by use corresponding numbers of the integral equations expressed in the form of dependence reaction force P_k of the corresponding rheological coordinates in the form (2) or (4) and with corresponding change of the coordinate from Descartes to generalized. Equations (7) are universal because their applications are possible in the case of the arbitrary number of the hereditary interactions between material particles in the system, and also in the sense when number of the hereditary interactions is larger then number of the system degree of freedom, $K > n$.

Applications of the Lagrange's equations second kind for hereditary systems in the form (7) with use of the rheological constitutive equations of the rheological interactions in the form (1) or (2) is possible for the case "dynamical defined systems" for which determinant satisfy the following condition $\left| b_{kj}(q) \right| \neq 0, \; K \leq n$. For these cases Lagrange's equations second kind is easy to solve with respect to the reactions P_k by use equations (1) or (3).

For description of properties of the dynamics of the hereditary system by use relaxational or rheological kernel (resolvent) , these kernels are expressed by exponential or fractional-exponential forms [1] . Description of the hereditary properties of the system by use differential forms (1) and integral form (2) and (3) with exponential kernels is equivalent. For the case of the fractional-exponential forms of the kernel (2) and (3) in the integral form corresponding equivalent differential forms not exist.

Equivalency of the hereditary interactions and reactive forces in the systems of the automatic control gives possibility to extend theory of the analytical dynamics of the hereditary systems to the mechanical systems with automatic control. For example, automat with transfer function presented in the following form

$$W(p) = \frac{b_0 + b_1 p + \ldots + b_n p^n}{1 + a_1 p + \ldots + a_n p^n} \tag{8}$$

present a hereditary interaction (1) between material particles of the mechanical system.

The Lagrange's mechanics of the hereditary systems is extended and generalized to the thermo-rheological [8] and piezo-rheological [9] mechanical systems.

Methods for solving problems of dynamics of the hereditary systems are considered with special Euler's Gama functions.

Approximation of the expressions for the coefficients of the damping and corresponding decrements as well as for frequency of the oscillations hereditary oscillatory systems are obtained with high accuracy.

Keywords: *Hereditary system, rheological element, rheological and relaxational kernels, standard hereditary element, integro-differential equation, material particles, rheonomic coordinate, rheological pendulum, rheonomic coordinate, covariant coordinate,* thermo-rheological and piezo-rheological hereditary elements.

Acknowledgements. Part of this research were supported by Ukrainian Higher Education Academy of Sciences and Kiev National University named Taras Shevchenko and also by the Ministry of Sciences, Technologies and Development of Republic Serbia trough Mathematical Institute SANU Belgrade Grants No. ON144002 Theoretical and Applied Mechanics of the Rigid and Solid Body. Mechanics of Materials, and also through Faculty of Mechanical Engineering University of Niš.

References

1. Работнов Ю.Н. *Элементы наследственной механики твердых тел*.–М.:Наука, 1977.–354 с.
2. Савин Г.Н., Рущицкий Я.Я. *Элементы механики наследственных сред*.–К.:Вища школа, 1975.-252 с.
3. Ржаницын А.Р. *Некоторые вопросы механики систем, деформирующихся во времени*.–М.:ГИТТЛ, 1949.–242 с.
4. Oleg Aleksandrovič Goroško (Ukrajina) i Katica (Stevanović) Hedrih (Srbija): *Analitička dinamika (mehanika) diskretnih naslednih sistema, (Analytical Dynamics (Mechanics) of Discrete Hereditary Systems), University of Ni{, 2001, Monograph, p*. 426 (in Serbian), YU ISBN 86-7181-054-2.
5. Hedrih (Stevanović) K., (1994), *The power of the rheonomic constraints change*, Zbornik radova JDM, Simpozijum iz Opšte mehanike, Novi Sad, 1994. pp. 177-185, .(in Serbian)
6. Goroshko, O.A., Puchko, N.P., *Lagrangian equations for the multibodies hereditary systems*, Facta Universitatis, Series Mechanics, Automatic Control and Robotics, Vol. 2, No 7, 1997, pp. 209-222.
7. Hedrih (Stevanović) K., *Rheonomic Coordinate method Applid to Nonlinear Vibration Systems with Hereditary Elements*, Facta Universitatis, Series, Mechanics, Automatic Control and Robotics, Vol. 2, No. 10, 2000, pp. 1111-1135. YU ISSSN 0354-2009.
8. Hedrih (Stevanović) K., *Thermorheological Hereditary Pendulum*, (Ref. No. TVP-11) Thermal Stresses 99, Edited by J.J. Skrzypek and R. B. Hetnarski, Cracow 1999, pp.199-202.
9. Katica (Stevanović) Hedrih, *Differential Equations of Two Mass Particles, Constrained with a Piezo-thermo-rheological Hereditary Element, Dynamics*, Proceedings of full papers, Edited by Dragutin Veličković, 5 th International Conference on Applied Electromagnetics, PES 2001., pp. 77-80.
10. Hedrih (Stevanović) K*., Discrete Continuum's Models and Thermo-rheological Elements - Basic Idea and Tensor Equations*, Proceedings of full papers, Edited by Dragutin Veličković, 6 th International Conference on Applied Electromagnetics, PES 2003, Stability Pact for South Eastern Europe, DAAD, pp. 127-130.
11. Hedrih (Stevanović) K., *Discrete Continuum's Models and Thermo-rheological Elements – Homogeneous Linear Chain and Plane/Space Material Nets,* Proceedings of full papers, Edited by Dragutin Veličković, 6 th International Conference on Applied Electromagnetics, PES 2003, Stability Pact for South Eastern Europe, DAAD, pp. 131-134..
12. Hedrih (Stevanović), K., (2007), *Energy analysis in the nonlinear hybrid system containing linear and nonlinear subsystem coupled by hereditary element**, Nonlinear Dynamics***, Springer, DOI 10.1007/s11071-007-9197-2
13. Hedrih (Stevanović) K., (2006), Modes of the Homogeneous Chain Dynamics, ***Signal Processing***, ***Elsevier,*** 86(2006), 2678-2702.. ISSN: 0165-1684 www.sciencedirect.com/science/journal/01651684