# Hybrid Systems and Hybrid Dynamics: Theory and Applications

Katica (Stevanović) Hedrih

*Faculty of Mechanical Engineering University of Niš, Mathematical Institute SANU*

*Yu-18 000 - Niš, ul. Vojvode Tankosića 3/22, Serbia and Montenegro, Yugoslavia, Telefax: 381 18 41 663 / Mobile 063 8 75 75 99*

*e-mail: katica@masfak.ni.ac.yu * e-mail(houm): khedrih@eunet.yu*

A survey as review of author's research result in area of dynamics of coupled subsystems and coupled dynamical processes into hybrid systems are presented.

The sets of two or many coupled partial as well as partial integro-differential equations, and also partial fractional differential equations of transversal vibrations of a elastically, as well as visco-elastic, and also creep connected double beam, as well as double plate systems, and also axially moving double belt system have been derived. The beam's as well as plate's materials, and also belt's materials are elastic or viscoelastic or creeping and constitutive relations of stress-strain states are expressed through members with integral parts or of the fractional order derivatives.

The analytical solutions of the sets of two coupled partial as well as partial integro-differential equations, and also partial fractional differential equations of corresponding dynamical free and forced processes are obtained by using method of Bernoulli's particular integral and Lalace's transform method asymptotic averaged method.

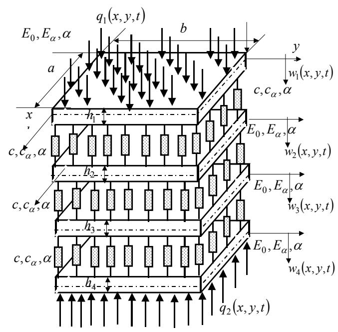
It is shown that to one mode vibrations correspond a infinite sets correspond the like or two-frequency regime for free vibrations induced by initial conditions. Analytical solutions show us that connection between beams, as well as between plates, and also between belts coursed appearance of like two-frequency regime of time functions correspond to one eigen amplitude function of one mode from infinite mode sets, and also that time functions of different the $n$-family or $mn$-family vibration modes $n, m = 1,2,3,4,....\infty$ are uncoupled.

The partial differential equations as well as partial integro differential equations of transversal stochastic vibration of a parametrically excited beam was derived. The influence of rotatory inertia of beam cross section and transverse shear of beam cross section under the transverse force, and the corresponding members in the partial differential equation are taken into account. Bernoulli particular integral method and Lagrange method of variation constant are used for the transformation problem. The asymptotic averaged method is used for obtaining the first approximation of Itô stochastic differential equations. The sets of Lyapunov exponents are obtained.

**Keywords:** *Double beam system, double plate system, double belt system, elastic, visco-elastic, creep, fractional order derivative, vibrations, partial integro-differential equations, partial fractional-order differential equation, stochastic Itô differential equations, Lyapunov exponents, multifrequency.*

***Acknowledgment:*** *Parts of this research were supported by the Ministry of Sciences, Technologies and Development of Republic Serbia trough Mathematical Institute SANU Belgrade and Faculty of Mechanical Engineering University of Niš Grants No. ON144002 Theoretical and Applied Mechanics of Rigid and Solid Body. Mechanics of Materials.*

# *Figures*



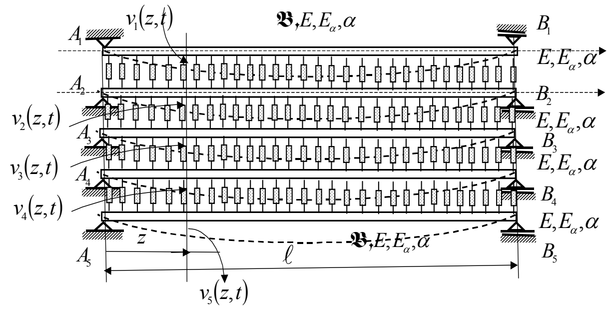


**Figure 1.** A creeping connected multi plate system  F**igure 2.** A creeping connected multi beam system**.**

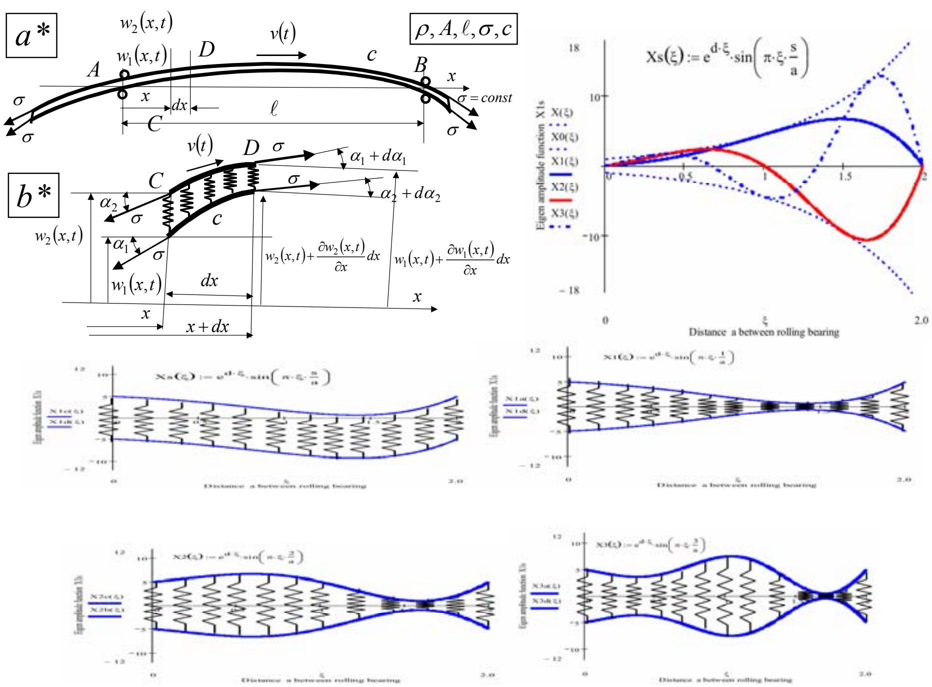



**Figure 3.** *Transversal vibrations of the axially moving sandwich belt system .*
a* *Kinetics parameters of the transversal vibrations of the axially moving sandwich belt.*
b* *Elementary segment of the axially moving sandwich belt with length* $dx$ *and notations of the kinetics parameters;* **c*** *Eigen amplitude function for first three modes of the double belt system vibrations,*
*Amplitude forms for transversal vibrations of the axially moving double sandwich belt system for some of possible cases: for first (d*) and (e*), for second( f*) and for third (g*) mode.*

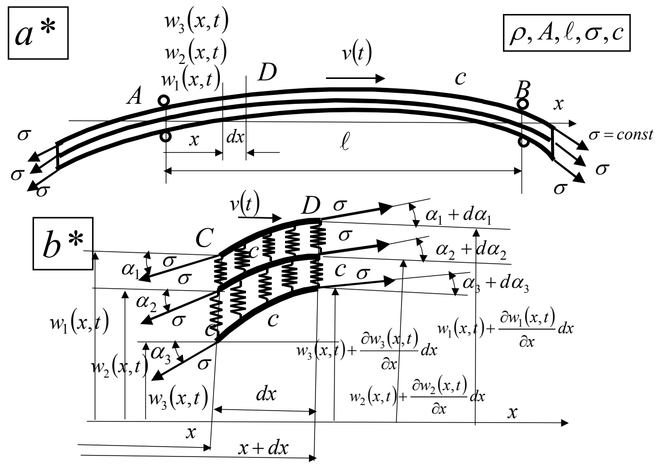



**Figure 4.** Transversal vibrations of the axially moving sandwich multi (three) belt system

a* Kinetics parameters of the transversal vibrations of the axially moving sandwich multi belt system

b* Elementary segment of the axially moving sandwich multi belt system with length $dx$ and notations of the kinetics parameters

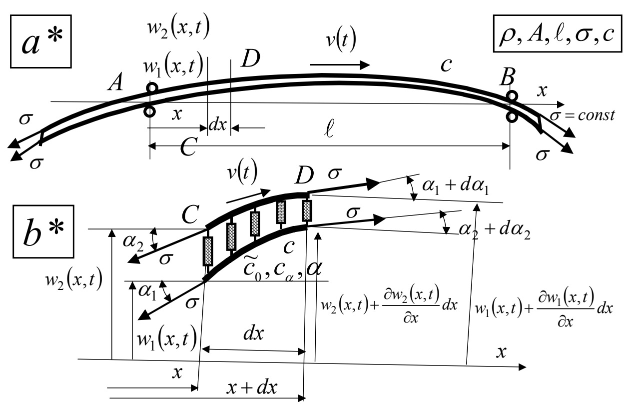



**Figure 5** Transversal vibrations of the axially moving sandwich double belt system with creep layer.

a* Kinetics parameters of transversal vibrations of the axially moving sandwich double belt system with creep layer.

b* Elementary segment of the axially moving sandwich double belt system with creep layer and with length $dx$ and notations of the kinetics parameters.

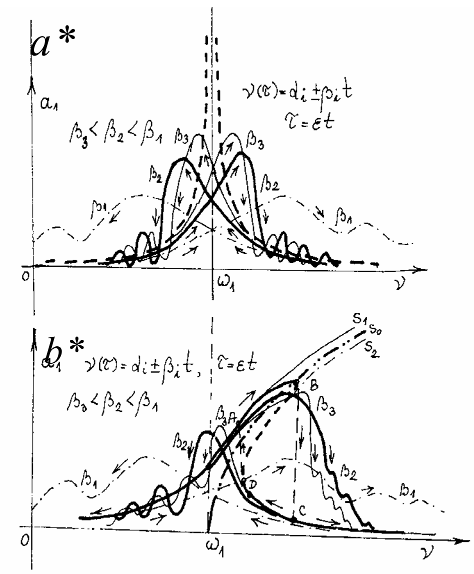


Figure 6.

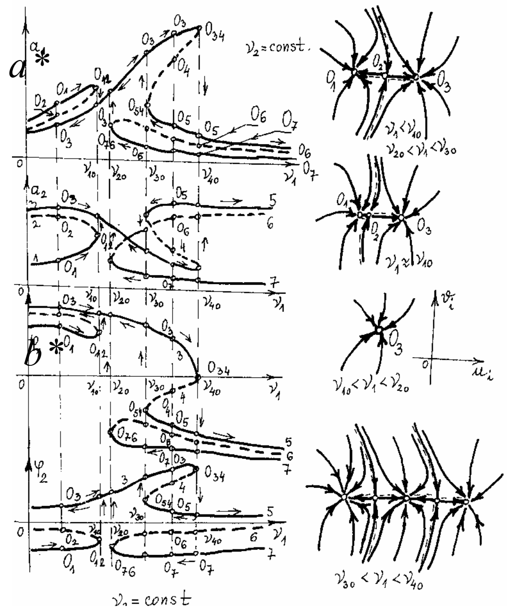


Figure 7.

**Figure 6.** Amplitude-frequency curves for stationary and nonstationary resonant regime for linear (**a***) and nonlinear (**b***) harmonic of an elastic body oscillatory process for different velocities of forced excitation frequency change passing through the resonant range corresponding to one frequency regimes of the elastic body vibrations..

**Figure 7.** Amplitude-frequency and phase-frequency curves of a stationary resonant state of two-frequency nonlinear oscillations of an nonlinear elastic body.

## *References*


[1] Hedrih (Stevanović) K., (2006), *Transversal Vibration of a Parametrically Excited Beam: Influence of Rotatory Inertia and Transverse Shear on Stochastic Stability of Deformable Forms and Processes*, ©Freund Publishing House Ltd. ***International Journal of Nonlinear Sciences and Numerical Simulation***, 7(1), 117-124, 2006. **Impact Factor 2,345 .**

[2] Hedrih (Stevanović) K., (2006), *Transversal Vibrations of Double-Plate Systems*, ***Acta Mechanica Sinica, Springer***, (2006) 22, pp. 487-501 (hard cower and on line). **ISI List**

[3] Hedrih (Stevanović) K., (2006), *Modes of the Homogeneous Chain Dynamics*, ***Signal Processing***, ***Elsevier,*** 86(2006), 2678-2702.. ISSN: 0165-1684 www.sciencedirect.com/science/journal/01651684 **ISI List**

[4] Hedrih (Stevanović) K., (2006), Vukota Babović, Dragan Šarković, (2006), An auxi liary size distribution model for the ultrasonically produced water droplets, ***Experimental Thermal and Fluid Science - ETF, Elsevier***, Volume 30, Issue 6, June 2006, Pages 559-564. http://www.elsevier.com/locate/etfs **ISI List**

[5] Hedrih (Stevanović), K., (2006), *Transversal vibrations of the axially moving double belt system with creep layer, Preprints, 2nd* ***IFAC*** *Workshop on Fractional Differentiation and its Applications*, 19-21 July, 2006, Porto, Portugal, pp.230-235. +CD. IFAC WS 2006 0007 PT, ISBN 972-8688-42-3, 978-972-8688-42-4.ISBN . http;/www.iser.ipp.pl

[6] Hedrih (Stevanović), K., (2006), *Measurements of Dynamical System Integrity and Fracture Mechanics*, Invited Key Lecture in ***Fracture of Nano and Engineering Materials and Structures*** (p. 1416), Edited by E.E.Gdoutos, Proceedings of the 16th *European Conference of Fracture* (p. 1416), ***Springer 2006***, pp. 793-794, plus CD 1-15. ISBN 1-4020-4971-4, BARCOD 9 781402 049712. http://ecf16.civil.duth.gr

[7] Booklet of Abstracts - *Minisymposium - Integrity of Dynamical Systems- ECF16*, Edited by Katica (Stevanović) Hedrih, invited organizer of Minisymposia ***IDS-ECF - the 16th European Conference of Fracture***, Greece 2006, Centre for Nonlinear Dynamics and Active Structures, Faculty of Mechanical Engineering University of Niš and Democritus University of Thrace, Xanthi, Greece, 2006. p. 324. ISBN 86-80587-57-5. BarCod 9 788680 587578. COBIS. SR - ID 130756620. UDK 531:01 /048); 530:182(048); 530:42 (048); 53:005.745 (100)

[8] Hedrih (Stevanović) K. and Simonović, J., (2006), *Characteristic Eigen Numbers and Frequencies of the Transversal Vibrations of Sandwich System*, SEECCM 06 –First South-East European Conference on Computational Mechanics, Proceedings, Editors – M. Kojic and M Papadrakakis, Kragujevac, pp. 90-94. + CD. hppt://www.seeccn.kg.ac.yu. ISBN 86-81037-13-7, COBISS.SR-ID 131758092, udc 531/533(082)

[9] Hedrih (Stevanović) K., (2005), Partial Fractional Order Differential Equations of Transversal Vibrations of Creep-connected Double Plate Systems, in Monograph - Fractional Differentiation and its Applications, Edited by Alain Le Mahaute, J. A. Tenreiro Machado, Jean Claude Trigeassou and Jocelyn Sabatier, **U-Book**, pp. 289-302.

[10] Hedrih (Stevanović) K., (2005), *Nonlinear Dynamics of a Heavy Material Particle Along Circle which Rotates and Optimal Control***, Chaotic Dynamics and Control of Systems and Processes in Mechanics** (Eds: G. Rega, and F. Vestroni), p. 37-45. IUTAM Book, **in Series *Solid Mechanics and Its Applications*, *Editerd by G.M.L. Gladwell, Springer***,. 2005, XXVI, 504 p., Hardcover ISBN: 1-4020-3267-6.
**http://www.springeronline.com/sgw/cda/frontpage/0,11855,5-40356-69-33113256-detailsPage%253Dbookseries%257Ctitles%257Ctitles%2526DISPLAY_TYPE%253DDISPLAY_TYPE_TITLES,00.html**
**http://www.springeronline.com/sgw/cda/frontpage/0,11855,5-40106-69-33113256-0,00.html?changeHeader=true**

[11] Hedrih (Stevanović) K., (2005), *Homoclinic Orbits Layering in the Coupled Rotor Nonlinear Dynamics and Chaotic Clock Models*, SM17 – Multibody Dynamics (M. Geraldin and F. Pfeiffer), p. Lxiii – CD - SM10624, Mechanics of the 21st Century (21st **ICTAM,** Warsaw 2004) - CD ROM INCLUDED, edited by Witold Gutkowski and Tomasz A. Kowalewski, IUTAM, **Springer 2005**, ISBN 1-4020-3456-3, Hardcover., p. 421+CD. ISBN-13 978-1-4020-3456-5 (HB), ISBN-10 1-4020-3559-4(e-book), ISBN-13-978-1-4020-3559-3
http://www.springeronline.com/sgw/cda/frontpage/0,11855,5-185-22-48265835-detailsPage%253Dppmmedia%257Ctoc%257Ctoc,00.html

[12] Hedrih (Stevanović) K., (2005)**,** *Integrity of Dynamical Systems, Journal Nonlinear Analysis***,** • **ARTICLE,** ***Nonlinear Analysis****, Volume 63, Issues 5-7, 30 November 2005-15 December 2005, Pages 854-871*
http://www.sciencedirect.com/science?_ob=QuickSearchListURL&_method=list&_aset=V-WA-A-W-A-MsSAYVA-UUW-U-AABZVEECDC-AABVUYUBDC-CWBCCAUAU-A-U&_sort=d&view=c&_st=13&_acct=C000053038&_version=1&_userid=1793222&md5=ceccc42b2422cce39c4f7934f5376895.

[13] Hedrih (Stevanović) K., *A Trigger of Coupled Singularities*, **MECCANICA**, Vol.39, No. 3, 2004., pp. 295-314. International Journal of the Italian Association of Theoretical and Applied Mechanics, Meccanica**,** Publisher: **Springer Science**+Business Media B.V., Formerly Kluwer Academic Publishers B.V. ISSN: 0025-6455 (Paper) 1572-9648 (Online) DOI: 10.1023/B:MECC.0000022994.81090.5f, Issue: Volume 39, Number 3, Date: June 2004, Pages: 295 - 314
http://www.springerlink.com/app/home/contribution.asp?wasp=ddefdd004a224ad7bd78087cafbb21c2&referrer=parent&backto=issue,6,12;journal,5,47;linkingpublicationresults,1:102958,1

[14] Hedrih (Stevanović) K., (2005), Partial Fractional Differential Equations of Creeping and Vibrations of Plate and their Solutions (First Part), Journal of the Mechanical Behavior of Materials, Freund Publishing House Ltd. 2005, pp. 305-314. **ISSN 0334-8938**
http://www.freundpublishing.com/JOURNALS/materials_science_and_engineering.htm

[15] Hedrih (Stevanović) K., The Transversal Creeping Vibrations Of A Fractional Derivative Order Constitutive Relation of Non-Homogeneous Beam, Journal Mathematical Problems in Engineering, Special Issue on Nonlinear Dynamics and Their Applications to Engineering Sciences, Hindawi Publishing Corp., http://www.hindawi.com/ 2006, pp. (in press).
http://www.hindawi.com/journals/mpe/forthcoming/S1024123X05501133.html
**ISSN: 1024-123X, e-ISSN: 1563-5147**

[16] Hedrih (Stevanović) K., (2005), Transveral vibrations of creep connected multi plate homoheneous systems, CD Proceedings, Fifth EUROMECH Nonlinear Dynamics Conference, Eindhoven University of Technology, Editors: D.H. van Campen, M.D. Lazurko, W.P.J.M. van den Over, ID of contribution 11-428, pp. 1445-1454. ISBN 90 386 2667 3, www.enoc2005.tue.nl

[17] Hedrih (Stevanović) K., (2005), Multifrequency forced vibrations of thin elastic shels, CD Proceedings, Fifth EUROMECH Nonlinear Dynamics Conference, Eindhoven University of Technology, Editors: D.H. van Campen, M.D. Lazurko, W.P.J.M. van den Over, ID of contribution 21-439, pp2417-2426. ISBN 90 386 2667 3, www.enoc2005.tue.nl

[18] Katica (Stevanović) Hedrih, (2005), Eigen Amplitude Vectors and Functions Extended Orthogonality of Small Oscillations Mixed Systems of the Coupled Discrete and Continuous Subsystems**,** Facta Universitatis, Series Mechanics, Automatic Control and Robotics, Vol. 4 No. 17, 2005. pp. 225-243. YU ISSN 0534-2009. UDC 534.1:534.012:534.013.
**http://facta.junis.ni.ac.yu/facta/macar/macar200501/macar200501-04.html**

[19] Katica (Stevanović) Hedrih, ***Transversal vibrations of the axially moving sandwich belts , Invited Lecture – 40 minutes, ISRAMES December 2-4, 2005. Kalyani, West Bengal, India.****The* ***International Symposium on "Recent Advances in Mathematical Sciences and Earth Sciences"*** *2nd to 4th December 2005. Department of Mathematics, University of Kalyani.*

[20] Katica (Stevanović) Hedrih: Measurements of dynamical systems integrity and fracture mechanics, Invited plenary lecture, ISUMEL 2005 Lviv, Ukraine, 7-й МІЖНАРОДНИЙ СИМПОЗІУМ УКРАЇНСЬКИХ ІНЖЕНЕРІВ-МЕХАНІКІВ У ЛЬВОВІ, *(МСУІМЛ — 7), 18 — 20 травня 2005 р., Присвячений 160-річчю Національного університету „Львівська політехніка”*

[20] Katica (Stevanović) Hedrih, (2006), Dynamics of a sandwich structures, Invited Plenary Lecture, CD Proceednigs of Full papers, Current Problems in Civil Engineering, Faculty od Civil Engineering Subotica, 2006.

`[21] Goroško O. A. i Hedrih (Stevanović) K., Analitička dinamika (mehanika) diskretnih naslednih sistema, *(*Analytical Dynamics (Mechanics) of Discrete Hereditary Systems*), University of Niš, 2001, Monograph, p*. 426, YU ISBN 86-7181-054-2.

[22] Hedrih (Stevanović) K., (2004), *Discrete Continuum Method, COMPUTATIONAL MECHANICS*, WCCM VI in conjunction with APCOM'04, Edited by Z.H.Zao, M.W.Zuang, W.X.Zhong © ***2004 Tsinghua University Press & Springer-Verlag,*** pp. 1-11, CD. IACAM International Association for Computational Mechanics – www. iacm.info, ISBN 7-89494-512-9.

[23] Hedrih (Stevanović) K., (2004), Creep Vibrations of a Fractional Derivative Order Constititive Relation Deformable Bodies, PACAM VIII. La Habana, 2004, ***Applied Mechanics in Americas***, Vol. 10. pp. 548-551, ISBN 959-7056-20-8.

[24] Hedrih (Stevanović) K., (2003), Frequency equations of small oscillations mixed systems of the coupled discrete and continuous subsystems, *Mehanika tverdogo tela* (Rigid Body Mechanics, Donetsk, UDC 531.1:534.012:534.013:, ISSN 0321-1975, vip. 33, pp. 174-189